Janice VanCleave's

MATH

for Every Kid

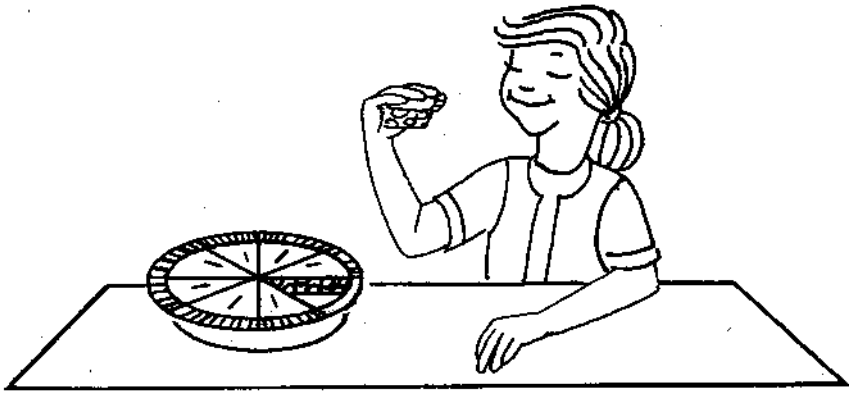
Easy Activities that Make Learning Math F

I
Basics

1
Fractions

Purpose To write fractions.

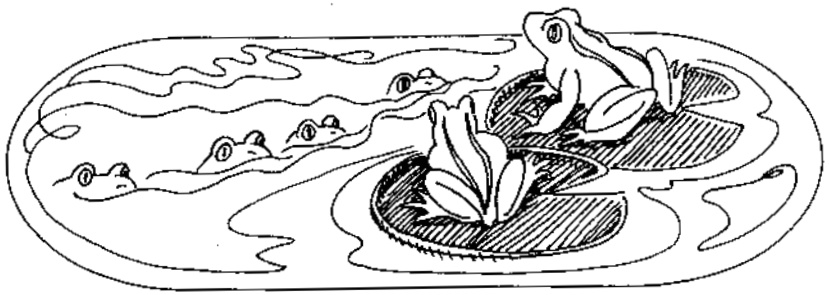
Facts A fraction tells how many parts are in the whole and refers to a part of the total.



The pie is divided into 8 equal parts. Sue is eating one of the pie pieces. $\frac{1}{8}$ is the fraction that tells what part of the pie Sue is eating. When you read a fraction, say the top number first and then the bottom number. $\frac{1}{8}$ is read as one-eighth.

The top number of a fraction is called the **numerator** and tells how many of the equal parts of the whole are being considered. The bottom number is called the **denominator** and tells the total number of equal parts in the whole.

$$\frac{1}{8} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{Pieces of pie being eaten}}{\text{Total number of pie pieces}}$$



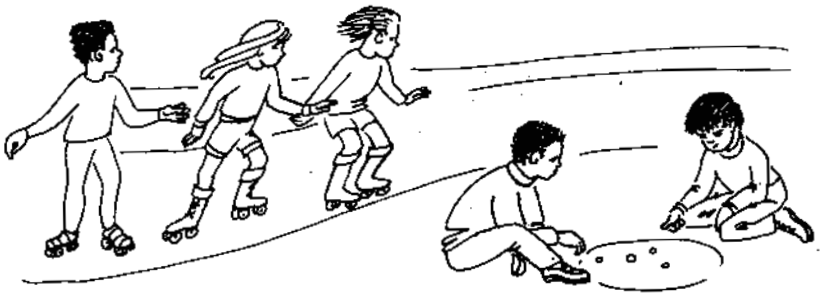
Problem

Question What fraction of the total number of frogs are in the water?

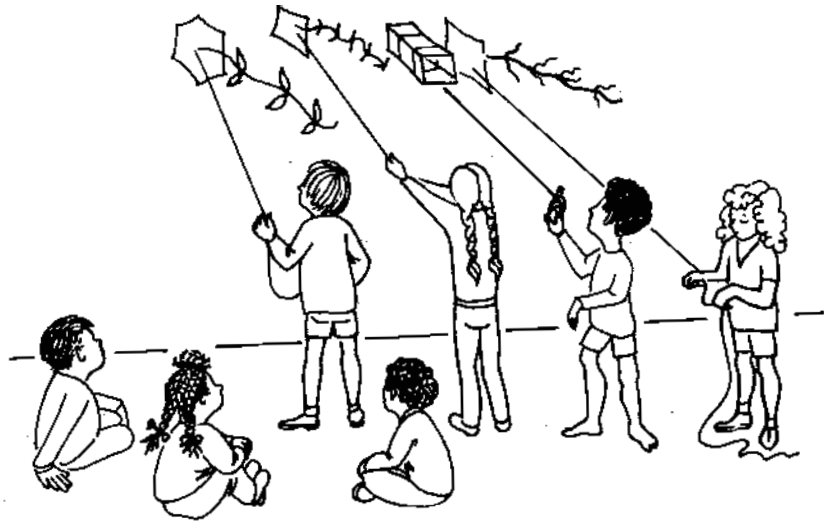
Think! How many frogs are in the water? 4
What is the total number of frogs present? 6
4 of the 6 frogs are in the water.

Answer 4/6 of the frogs are in the water.

Exercise



1. **a.** What fraction of the children are skating?
   b. What fraction of the children are playing marbles?



2. **a.** What fraction of the children are flying kites?
   b. What fraction of the children are sitting?

Activity: HALF-LIFE

Purpose To demonstrate how radioactive materials change.

Materials

- typing paper
- 2 empty shoe boxes
- timer
- marking pen
- scissors

Procedure

- Use the marking pen to label one shoe box Changed and the other box Unchanged.
- Use the scissors to cut the sheet of paper in half.
- Place one of the paper halves in the box marked Unchanged.
- Place the second half piece of paper in the Changed box. All papers placed in the Changed box are to be left there, undisturbed, throughout the experiment.

2
Fractional Parts

Purpose To find the fractional parts.

Facts When determining the fractional part of a number follow these steps:

Step 1 Write any whole numbers as fractions by placing the number over 1.

Example

$$12 = \frac{12}{1}$$

Step 2 Multiply the numerators (the top numbers) and denominators (the bottom numbers) of the two fractions.

Examples

$$\frac{2}{3} \times \frac{12}{1} = \frac{2 \times 12}{3 \times 1} = \frac{24}{3}$$

$$\frac{3}{8} \times \frac{2}{4} = \frac{3 \times 2}{8 \times 4} = \frac{6}{32}$$

Step 3 Reduce the fraction to its simplest form.

Example When the numerator is larger than the denominator as in $^{24}/_{3}$, the fraction can be reduced by dividing the numerator by the denominator.

$$3\overline{)24}\quad 8,\ 24,\ 0$$

If the denominator does not evenly divide into the numerator, as in the number $^{7}/_{3}$, express the remainder as a fraction.

$$3\overline{)7}\quad 2,\ 6,\ 1 \longleftarrow \text{remainder}$$

Answer $2\,^{1}/_{3}$

Example When the numerator is smaller than the denominator, as in $^{6}/_{32}$, divide the numerator and denominator by the largest **common factor**. A common factor is one number that will divide evenly into both the numerator and denominator.

$$\frac{6 \div 2}{32 \div 2} = \frac{3}{16}$$

Problems

Question 1 Carol spends $^{1}/_{12}$ of each day studying. How many hours a day does this equal?

Think! There are 24 hours in one day.

$$\frac{1}{12} \times 24 \text{ hours}$$

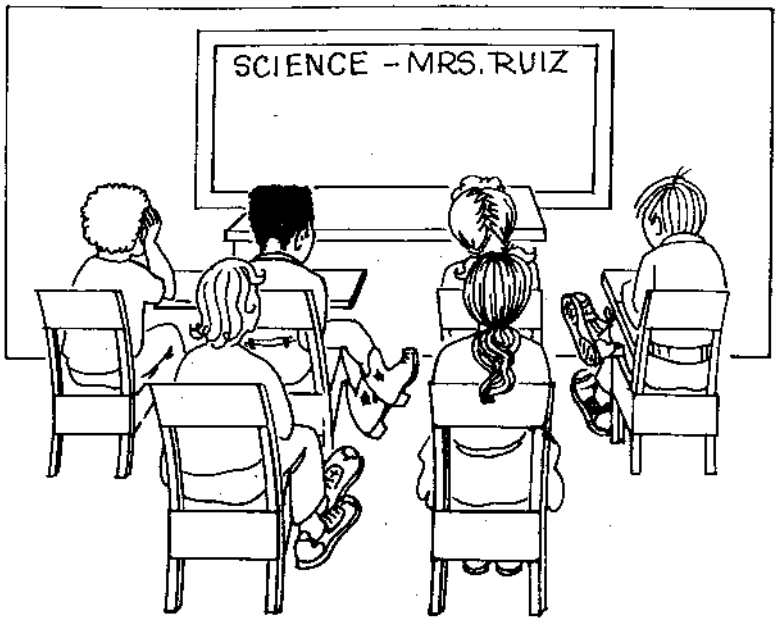
Step 1 $$24 = \frac{24}{1}$$

Step 2 $$\frac{1}{12} \times \frac{24}{1} = \frac{1 \times 24}{12 \times 1} = \frac{24}{12}$$

Step 3 $$12\overline{)24}\quad 2,\ 24,\ 0$$

Answer 2 hours

Question 2 1/2 of Mrs. Ruiz's science class are boys. 2/3 of the boys in the class wear tennis shoes. What part or fraction of the class is made up of boys wearing tennis shoes?



Think! $\frac{1}{2} \times \frac{2}{3}$ = Number of boys wearing tennis shoes.

Step 1 $\frac{1 \times 2}{2 \times 3} = \frac{2}{6}$

Step 2 $\frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

Answer 1/3 of the class consists of boys wearing tennis shoes.

Exercise

1. Patsy read 40 books during the month of August. Of these, 3/4 were mystery books. How many mystery books did she read in August?
2. If 3/5 of the 60 seeds planted by Wade grew, what was the total number of plants in his garden?
3. Amber spends 1/4 of each day sleeping. The amount of time that she sleeps during one year converts to how many days?
4. 3/10 of the earth is land. North America makes up 1/6 of the land area. What part of the land surface of the earth is North America?

Activity: MIXTURE

Purpose To demonstrate the fractional parts of air.

Materials

- 78 miniature marshmallows
- 21 red gumdrops
- 1 black gumdrop
- 1 resealable plastic bag, 1 qt size

Procedure

- Place the marshmallows and gumdrops in the plastic bag.
- Close the bag and shake thoroughly to mix.
- Dip your hand inside the plastic bag and scoop up some materials.

■ Count the number of marshmallows, red gumdrops, and black gumdrops in the sample taken from the bag.

Results There will be fewer red gumdrops than there are marshmallows in any sample taken from the bag. The black gumdrop is rarely picked up.

Did You Know?

The mixture represents a sample of clean, dry air, which contains 78/100 parts nitrogen (the marshmallows), 21/100 parts oxygen (the red gumdrops), and 1/100 part other gases (the black gumdrop). Samples of air taken from various places on the earth vary only slightly in composition.

Solutions

1. ***Think!*** $\frac{3}{4} \times 40 = ?$

Step 1 $\frac{3 \times 40}{4 \times 1} = \frac{120}{4}$

Step 2
$$4\overline{)120}\quad 30,\ 12,\ 000$$

Answer Patsy read 30 mystery books during August.

2. ***Think!*** $\frac{3}{5} \times 60 =$ Number of plants

Step 1 $\frac{3 \times 60}{5 \times 1} = \frac{180}{5}$

Step 2
$$5\overline{)180}\quad 36,\ 15,\ 30,\ 30,\ 00$$

Answer 36 plants

3. ***Think!*** 1 year has 365 days

$\frac{1}{4} \times 365 = \frac{1}{4} \times \frac{365}{1}$

Step 1 $\frac{1 \times 365}{4 \times 1} = \frac{365}{4}$

Step 2
$$4\overline{)365}\quad 91,\ 36,\ 005,\ 4,\ 1 \longleftarrow \text{remainder}$$

Answer Amber sleeps enough hours each year to equal 91 1/4 days.

4. ***Think!*** $\frac{3}{10} \times \frac{1}{6} = ?$

Step 1 $\frac{3 \times 1}{10 \times 6} = \frac{3}{60}$

Step 2 $\frac{3 \div 3}{60 \div 3} = \frac{1}{20}$

Answer North America makes up 1/20 of the earth's land surface.

3
Equivalents

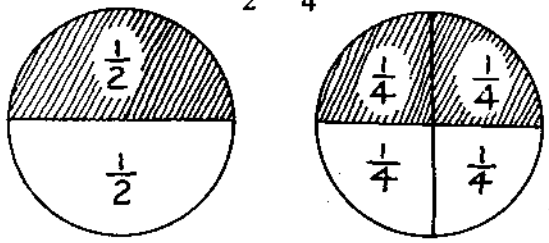
Purpose To write equivalent fractions.

Facts Equivalent fractions represent the same amount of a whole or group. 1/2 of a circle is the same amount as 2/4 of the same circle. This is expressed as: 1/2 = 2/4. It is just as true to say that 2/4 = 1/2. When changing a fraction with a small denominator to one with a larger denominator, multiply the numerator and denominator by the same number. To change a fraction with a large denominator to one with a smaller denominator, divide the numerator and denominator by the same number.

Problems

Question 1 Make the fractions equivalent.

$$\frac{1}{2} = \frac{?}{4}$$



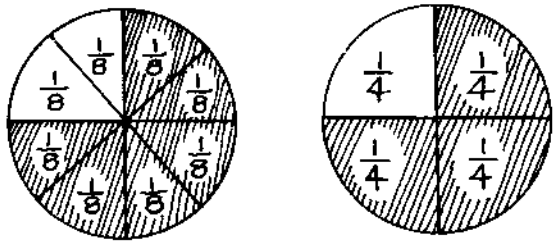
Think! $2 \times ? = 4$

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Answer $\frac{1}{2} = \frac{2}{4}$

Question 2 Make the fractions equivalent.

$$\frac{6}{8} = \frac{?}{4}$$



Think! $8 \div ? = 4$

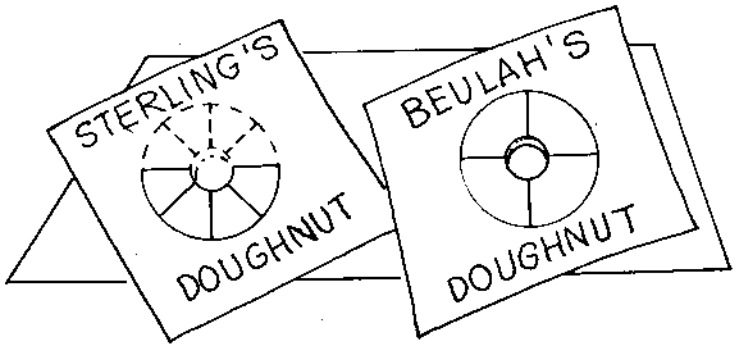
$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Answer $\frac{6}{8} = \frac{3}{4}$

Exercises

1. Sterling ate 4/8 of a doughnut. Beulah's donut is cut into four parts. How many parts must Beulah eat to equal what Sterling ate?

$$\frac{4}{8} = \frac{?}{4}$$



2. Tina cut a birthday cake into 16 equal pieces. Each person ate 1 piece of cake. How many people were served if 3/4 of the cake was eaten?

$$\frac{3}{4} = \frac{?}{16}$$

***3.** Lauren uses 100 coins to teach fractions to her sister Lacey. Determine how many pennies each fractional part of 100 represents.

a. $\frac{3}{4}$ of 100

b. $\frac{4}{25}$ of 100

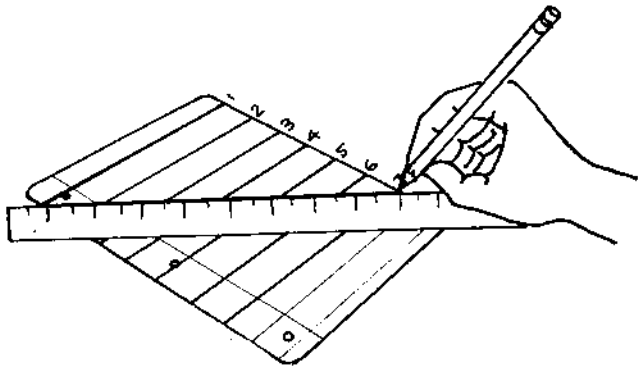
Activity: ONE LESS

Purpose To demonstrate that equivalent fractions represent the same amount.

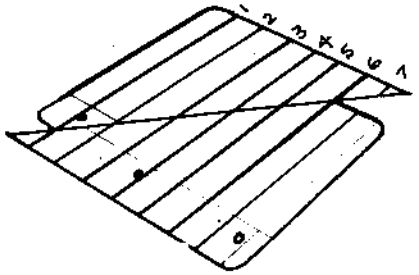
Materials 1 sheet of lined notebook paper
pencil
ruler
scissors

Procedure

- Lay the ruler along the top line of the paper.
- Start at the left margin and make a 6-in. (15-cm) mark over the line on the lined notebook paper.
- Move to the next line and make another 6-in. (15-cm) mark.
- Repeat until 7 separate marks are made.
- Place the ruler diagonally across the lines so that the ruler's edge touches the left end of the top line and the right end of the bottom line.



- Draw along the edge of the ruler, extending the lines to the edges of the paper.
- Use the scissors to cut across the diagonal line.
- Keep the paper pieces flat on a table and slide the right piece down to form 6 straight lines.
- Measure the length of each line.



Results Each of the 7 lines is 6 in. (15 cm) long. Shifting the paper pieces causes one of the lines to disappear and produces 6 lines that are each 7 in. (17.5 cm) long. The sum of the length of the 7 lines is 42 in. (105 cm), which is equivalent to the sum of the length of the 6

lines. The 6 separate parts are equivalent to the 7 separate parts and can be expressed in fraction form as $6/6 = 7/7$.

Did You Know?

You spend about 1/4 of your life sleeping. This is equal to 2,190 hours each year. To determine how many hours you have slept thus far in your life, multiply your age by 2,190. There are 8,760 hours in a year. What fraction of that time do you spend studying math?

Solutions

1. ***Think!*** $8 \div ? = 4$

$$\frac{4 \div 2}{8 \div 2} = \frac{2}{4}$$

$$\frac{4}{8} = \frac{2}{4}$$

Answer Beulah must eat 2 parts.

2. ***Think!*** $4 \times ? = 16$

$$\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$$

$$\frac{3}{4} = \frac{12}{16}$$

Answer 12 people were served.

3. a. ***Think!*** $4 \times ? = 100$

$$\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$$

$$\frac{3}{4} = \frac{75}{100}$$

Answer 75 pennies

3. b. ***Think*** $25 \times ? = 100$

$$\frac{4 \times 4}{25 \times 4} = \frac{16}{100}$$

$$\frac{4}{25} = \frac{16}{100}$$

Answer 16 pennies

3. Matthew counted his food calories for 1 week. What was his average daily calorie intake for the week?

Matthew

Day	Calorie Intake
Monday	1200
Tuesday	1300
Wednesday	1500
Thursday	1200
Friday	1800
Saturday	2200
Sunday	2000

4. The average age of Stella Cathey's children is 22 years. What is Carol's age?

Age of Stella's Children

Children	Age (Years)
Jim	23
Frances	24
Carol	?
Group Average	22

Activity: HOW LONG?

Purpose To determine the average length of a peanut.

Material 20 peanuts in the shell
ruler

Procedure

- Measure and record the length of each peanut to the nearest inch (centimeter).
- Add the 20 length measurements.

- Divide the sum of the length measurements by the number of peanuts, 20.

Results The sum of the lengths divided by the total number of peanuts measured gives the average length of the peanuts measured. Peanuts generally range from 1 to 2 in. (2½ to 5 cm) in length.

Did You Know?

There are about 250 average-sized peanuts in 1 pound (454 g) of peanuts. Peanuts are one of the most nutritious of all vegetables. There is more protein in a pound of peanuts than in a pound of meat.

Solutions

1. Step 1 Find the sum of the students:

$$95 + 96 + 100 + 101 + 102 + 97 + 95 = 686$$

Step 2 Divide the sum of the students by the total number of days, 7:

$$\begin{array}{rl} & 98 \longleftarrow \text{Average attendance} \\ \text{Number of days} \longrightarrow & 7\overline{)686} \longleftarrow \text{Sum of students} \\ & \underline{63} \\ & 56 \\ & \underline{56} \\ & 0 \end{array}$$

Answer 98 is the average attendance.

2. Step 1 Find the sum of the scores:

$9.8 + 9.2 + 9.9 + 9.7 + 10.0 = 48.6$

Step 2 Divide the sum by the total number of scores, 5:

$$\begin{array}{rl} & 9.72 \longleftarrow \text{Average} \\ \text{Number of scores} \longrightarrow & 5\overline{)48.60} \longleftarrow \text{Sum of scores} \\ & \underline{45} \\ & 36 \\ & \underline{35} \\ & 10 \\ & \underline{10} \\ & 0 \end{array}$$

Answer 9.72

3. Step 1 Find the sum of the calories:

$1200 + 1300 + 1500 + 1200 + 1800 + 2200 + 2000 = 11{,}200$

Step 2 Divide the sum of the calories by the number of days, 7:

$$\begin{array}{rl} & 1600 \longleftarrow \text{Average} \\ \text{Number of days} \longrightarrow & 7\overline{)11200} \longleftarrow \text{Sum of calories} \\ & \underline{7} \\ & 42 \\ & \underline{42} \\ & 0 \end{array}$$

Answer 1600 calories

4. ***Think!*** $3 \times 22 = 66$

Think! You now know that the sum of the three ages is:
$23 + 24 + ? = 66$

Think! The sum of $23 + 24 = 47$, therefore:
$47 + ? = 66$

Answer $? = 19$
Carol is 19 years old.

5
Multiples

Purpose To multiply whole and decimal numbers.

Facts The numbers that are multiplied together are called **factors** and the answer of the multiplication is the **product**. Multiply factors with decimals as you would whole numbers, placing the decimal in the product. The number of decimal places in the product is equal to the sum of the decimal places in all of the factors.

Example

Factor		Factor		Product
2.2	×	1.81	=	3.982
1 place	+	2 places	=	3 places

When multiplying three or more factors, multiply the first two numbers and then multiply the product by the next factor. Continue this until all the factors are used.

Example

$4 \times 3 \times 2 \times 5 =$
$4 \times 3 = 12$
$12 \times 2 = 24$
$24 \times 5 = 120$
or
$4 \times 3 \times 2 \times 5 = 120$

When the factors have more than one number, work with one number at a time.

Example

3.2	1 decimal place
×4.5	1 decimal place
160	Product of 3.2×5
+ 128	Product of 3.2×4
14.40	2 decimal places

Problem

Question Multiply $1.23 \times 0.81 \times 4$

Think! Multiply the first two factors:

1.23	2 decimal places
×0.81	2 decimal places
123	Product of 1.23×1
984	Product of 1.23×8
9963	

Think! Multiply the product, 9963, by 4:

9963
× 4
39852

Think! What is the sum of the decimal places in the three factors? 4

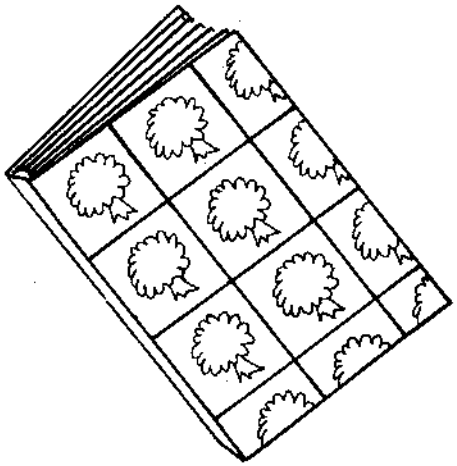
Answer 3.9852

Exercises

1. Charlotte can run one lap around the school track in 1.45 minutes. To determine how long it would take her to run 1.5 laps, multiply 1.45×1.5.

2. Lacey ate 2.5 cookies. Every cookie contained 4.5 raisins. To determine the total number of raisins eaten by Lacey, multiply 2.5×4.5.

3. Diane wants to cover her biology folder with stickers of trees. She needs 2.25 stickers to cover the width and 3.5 stickers to cover the length. To determine the number of stickers needed to cover the front and back of the notebook, multiply $2.25 \times 3.5 \times 2$.



*4. Would you take this job? A job pays $.01 the first day. If the pay doubles each day, the money received on day two would be twice the pay received on the previous day, or $2 \times \$.01 = \$.02$. Calculate the money received each day for 30 days starting with day 1 = $.01.

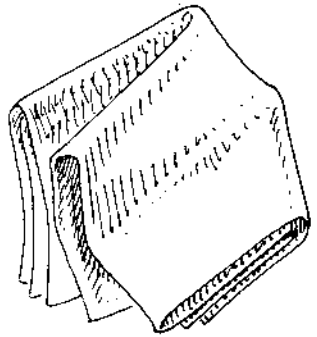
Activity: DOUBLING

Purpose To determine the number of sections formed by folding a sheet of paper a specific number of times.

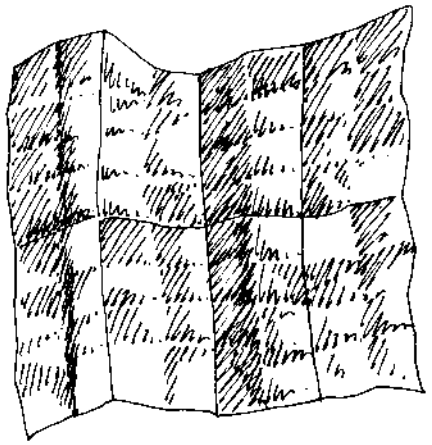
Materials typing paper
newspaper

Procedure

- Fold the typing paper in half to produce 2 sections.
- Fold the paper in half again to produce 4 sections.
- Continue to fold the paper until 6 folds have been made.
- Determine the number of sections produced by doubling the section number after each folding.



- Open the paper after the sixth folding and count the sections to check your calculated answer.



- Refold the typing paper and determine how many times it can be folded in half.
- Use the newspaper sheet and determine how many times it can be folded in half.

Results Six folds produce 64 sections. It is difficult to fold any size paper more than six times because of the thickness of the paper. The seventh fold produces 128 sections and an eighth fold would again double the number of sections, forming 256 sections.

Solutions

1.

$$\begin{array}{rl} 1.45 & \longleftarrow \text{2 decimal places} \\ \times\ 1.5 & \longleftarrow \text{1 decimal place} \\ \hline 725 & \\ 145 & \\ \hline 2.175 & \longleftarrow \text{3 decimal places} \end{array}$$

Answer 2.175 minutes to run 1.5 laps

2.

$$\begin{array}{rl} 2.5 & \longleftarrow \text{1 decimal place} \\ \times\ 4.5 & \longleftarrow \text{1 decimal place} \\ \hline 125 & \\ 100 & \\ \hline 11.25 & \longleftarrow \text{2 decimal places} \end{array}$$

Answer 11.25 raisins (*Note:* This is the same as 11 1/4.)

3.

$$\begin{array}{rl} 2.25 & \longleftarrow \text{2 decimal places} \\ \times\ 3.5 & \longleftarrow \text{1 decimal place} \\ \hline 1125 & \\ 675 & \\ \hline 7875 & \\ \times\ 2 & \\ \hline 15.750 & \longleftarrow \text{3 decimal places} \end{array}$$

Answer 15.750 stickers (*Note:* This is the same as 15 3/4.)

***4.**

Day	Salary ($)	Day	Salary ($)
1	.01	10	5.12
2	.02	11	10.24
3	.04	12	20.48
4	.08	13	40.96
5	.16	14	81.92
6	.32	15	163.84
7	.64	16	327.68
8	1.28	17	655.36
9	2.56	18	1310.72
19	2621.44	25	167,772.16
20	5242.88	26	335,544.32
21	10,485.76	27	671,088.64
22	20,971.52	28	1,342,177.30
23	41,943.04	29	2,684,354.60
24	83,886.08	30	5,368,709.20

Note that each day's salary was twice as much as the day before. A total amount of money received for any number of days can be quickly calculated by doubling the pay for the last day of work, then subtracting 1 from the product. Example: How much money was received during 4 days?

$$\begin{array}{rl} \text{Salary for day 4} & = \$.08 \\ & \times\ 2 \\ \hline & \$.16 \end{array}$$

Product − 1 = Total money received

$\$.16 - 1 = \$.15$ = Total money for 4 days

To check your answer, add up the money received during the four days.

$\$.01 + \$.02 + \$.04 + \$.08 + \ = \$.15$

The total amount of money received during the 30 days:

$\$5,368,709.20 \times 2 = \$10,737,418.40$

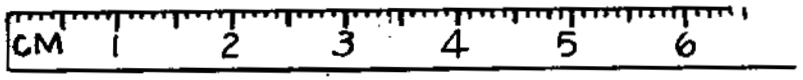
$\$10,737,418.40 - 1 = \$10,737,418.39$

II
Measurements

6
Centimeters

Purpose To use a metric ruler to measure lengths in centimeter units.

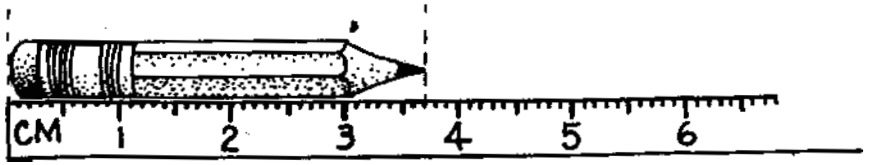
Facts On a metric ruler, the printed numbers indicate centimeter measurements. Each small division between the numbers is equal to 0.1 cm.



Problem

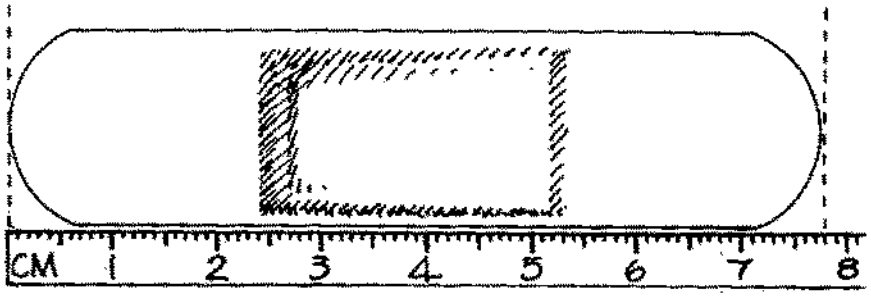
Question Determine the length of the pencil in centimeters. Express the answer to the nearest 0.1 cm.

Answer The length of the pencil is 3.7 cm.

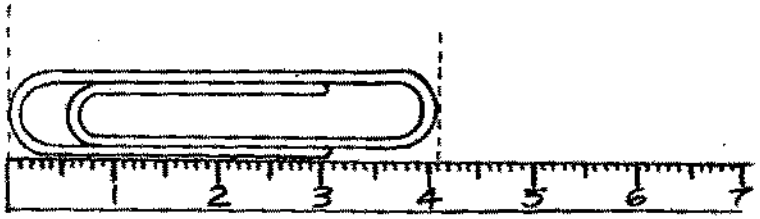


Exercises

1. What is the length of the adhesive bandage?



2. How long is the paper clip?



Activity: SPAN

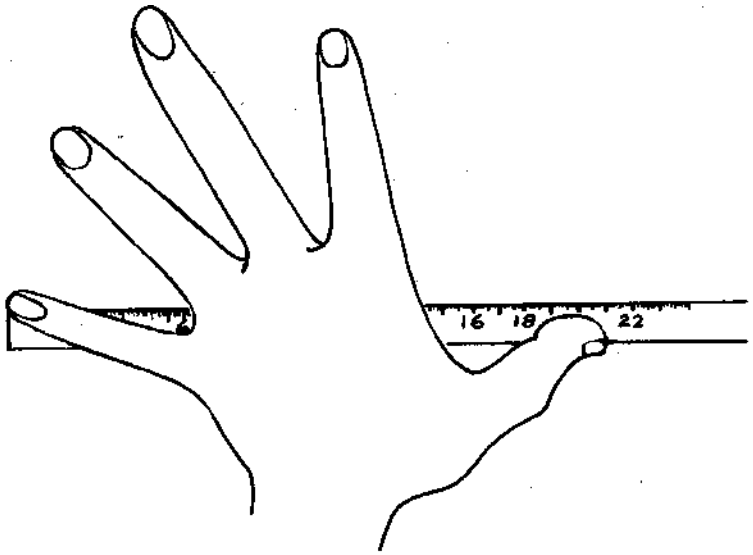
Purpose To measure lengths in spans and centimeters.

Materials

- hand
- metric ruler
- table (the kitchen table will work well)
- masking tape
- pencil

Procedure

- Stretch out the fingers on your left hand.
- Lay your stretched hand on top of the ruler, placing your little finger on the end of the ruler and stretching your thumb as far up the ruler as it will go.



- Record the length of your stretched hand to the nearest whole centimeter.
- Place a strip of tape along the longest edge of the table.
- Lay your stretched hand at the left end of the table edge on top of the tape.
- Mark the end of your thumb with the pencil.
- Move your stretched hand to the right, placing the end of your little finger on the pencil mark.
- Again mark the end of your thumb.
- Continue to move your hand across the edge of the table until the entire length of the table has been measured.
- If the last measurement is shorter than your stretched hand, count it only if the distance is more than half the distance across your stretched hand.
- Your stretched hand is called a **span.** Count the number of marks made and record the length of the table in spans.
- Multiply the number of spans times the centimeter length of your stretched hand to determine the length of the table in centimeters.

Results The number of marks will depend on the length of the table and how long the span of your hand is. If you remember the length of your hand span, it will give you a handy way to estimate lengths.

Did You Know?

Our measuring system was originally based on human measurements such as the span of your hand. A mile was equal to 1,000 steps of a Roman soldier and a yard was the length from the king's nose to the end of his thumb. Since the king was not always available, each person used his or her own outstretched arm to measure a yard.

The differences in the sizes of human bodies led to the need for a more standard way of measuring. In 1791, French scientists created the metric system of measurements. One ten-millionth of the distance from the North Pole to the Equator was scratched on a metal rod. Copies of the metal rod were made and used to measure metric lengths. With technological advancements, more precise measurements are possible. The distance that light travels in 1/299,792,458 of a second is now the standard meter length.

Solutions

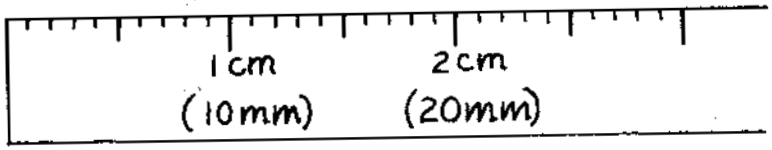
1. The length of the adhesive bandage is 7.8 cm.

2. The length of the paper clip is 4.1 cm.

7
Millimeters

Purpose To use a metric ruler to measure length in millimeter units.

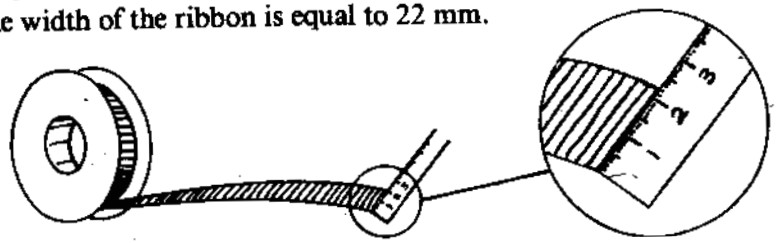
Facts On a metric ruler, the printed numbers indicate centimeter measurements. Each small division between the numbers is equal to 0.1 centimeters, which is the same as 1 millimeter. One centimeter equals 10 millimeters. Multiply each numbered centimeter measurement by 10. The symbol for the millimeter unit is mm.



Problem

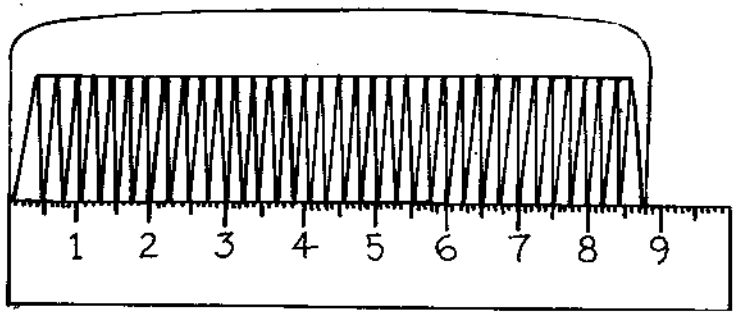
Question What is the width of the ribbon in millimeter (mm) units?

Answer The printed numbers on the ruler times 10 give a millimeter measurement. Each small division between the printed numbers equals 1 millimeter measurement. The edge of the ribbon stops at the second millimeter mark after the printed number 2 on the ruler. The width of the ribbon is equal to 22 mm.

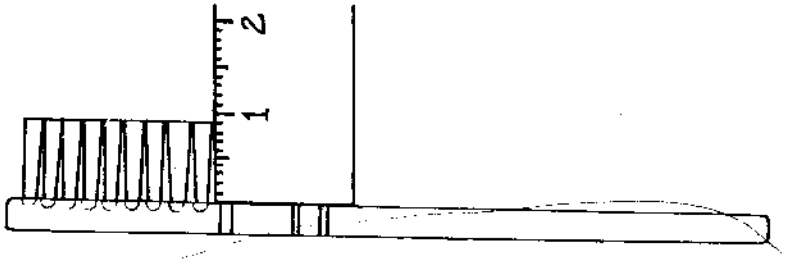


Exercises

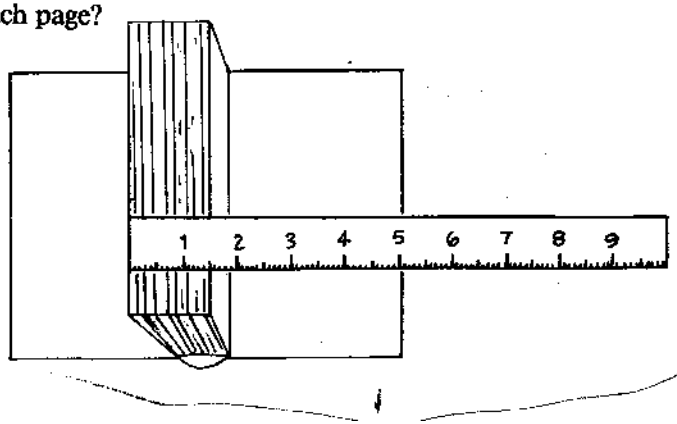
1. What is the length of the comb in millimeters?



2. How tall are the bristles on the toothbrush in millimeters?



3. There are 100 pages in the book. What is the thickness in millimeters of each page?



Activity: METRIC TAPE

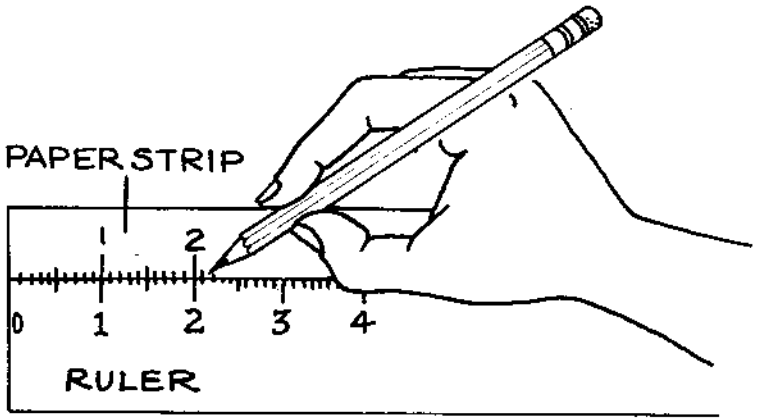
Purpose To make and measure millimeter lengths with a paper metric tape.

Materials

typing paper
scissors
marking pen
large chicken egg

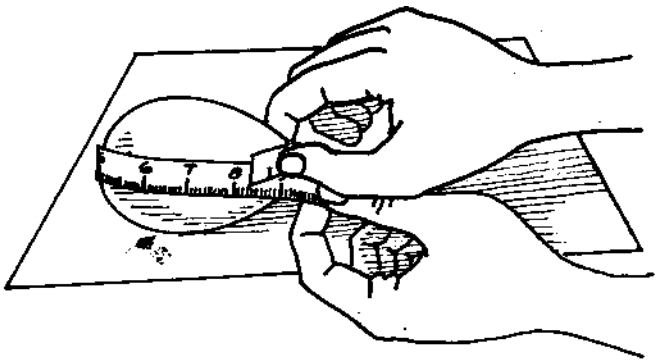
Procedure

- Measure and cut a 30 mm × 280 mm paper strip.
- Use the pencil to write ZERO across one end of the paper strip.
- Lay the ruler on the paper strip and use the pencil to mark the millimeter positions on the paper. Begin on the very edge of the paper strip marked ZERO.



- Use the paper measuring tape you just made to measure a large chicken egg from end to end.

Results The paper tape easily bends around objects, making it useful when measuring curved materials. The length of a chicken egg from end to end varies with each egg. The author's large egg measured 83 mm from end to end. Measure different-sized eggs and compare their measurements.



Did You Know?

The smallest bird egg is laid by the Vervain hummingbird of Jamaica. The egg measures about 9.9 mm in length (from end to end.

Solutions

1. The comb is 88 mm long.

2. The toothbrush bristles are 9 mm long.

3. The thickness of 100 pages is 15 mm. To determine the width of one page, divide 15 mm by 100.

$$
\begin{array}{r}
.15 \\
100\overline{)\,15.00} \\
\underline{100} \\
500 \\
\underline{500} \\
000
\end{array}
$$

The thickness of one page is 0.15 mm.

8
Perimeter

Purpose To find the perimeter of polygons.

Facts The **perimeter** is the distance around an object and can be found by adding the lengths of all sides. **Polygons** have straight sides that meet to form angles.

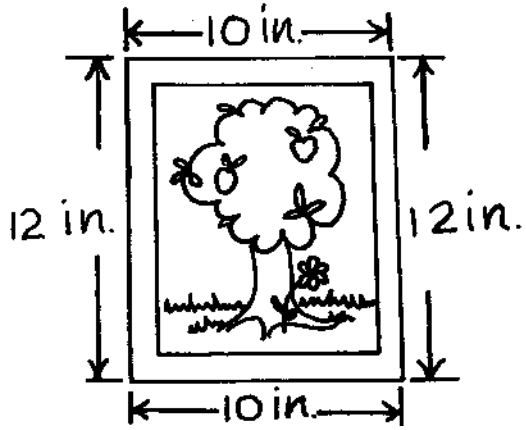
Problems

Question Determine the perimeter of each object.

1. **a.** The perimeter of the rectangular picture frame can be determined by adding together the length of all four sides:

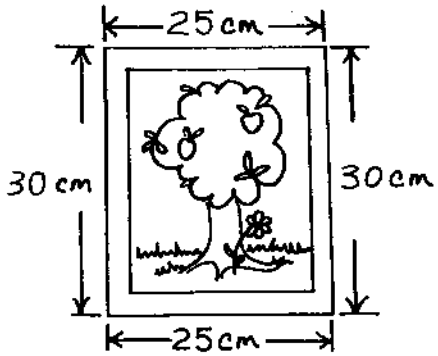
English

10 in. + 12 in. + 10 in. + 12 in. = 44 in.



Metric

25 cm + 30 cm + 25 cm + 30 cm = 110 cm



or

Determine the distance halfway around the rectangle and multiply by two:

English

Step 1 10 in. + 12 in. = 22 in.
Step 2 22 in. × 2 = 44 in.

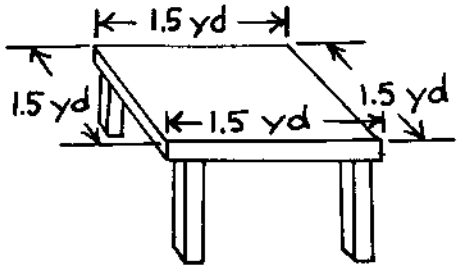
Metric

Step 1 25 cm + 30 cm = 55 cm
Step 2 55 cm × 2 = 110 cm

2. b. The perimeter of the square table top can be determined by adding together the length of all four sides:

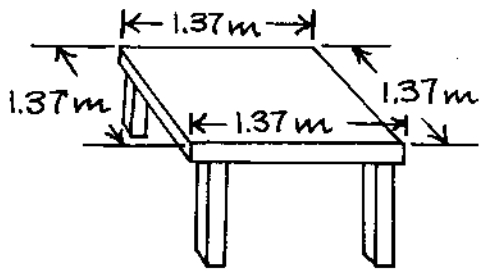
English

1.5 yd + 1.5 yd + 1.5 yd + 1.5 yd = 6.0 yd



Metric

1.37 m + 1.37 m + 1.37 m + 1.37 m = 5.48 m



Since all four sides are the same length, multiply the measurement of one of the sides by four:

English

1.5 yd × 4 = 6.0 yd

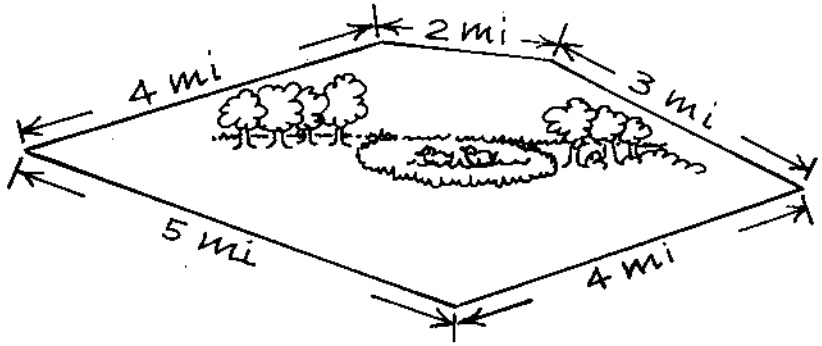
Metric

1.37 m × 4 = 5.48 m

3. c. The perimeter of this irregularly shaped park can be determined by adding all the sides together:

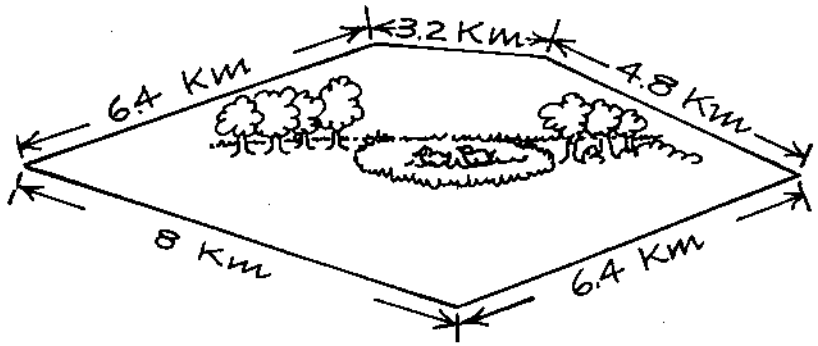
English

3 mi + 2 mi + 4 mi + 5 mi + 4 = 18 mi



Metric

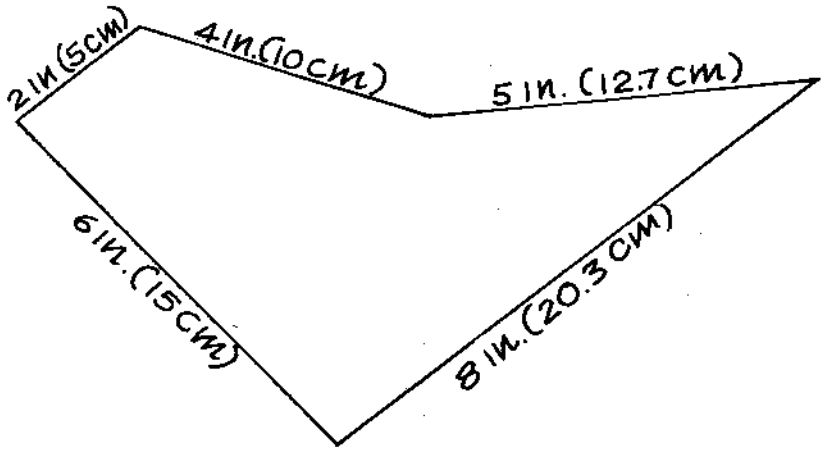
4.8 km + 3.2 km + 6.4 km + 8 km + 6.4 km = 28.8 km



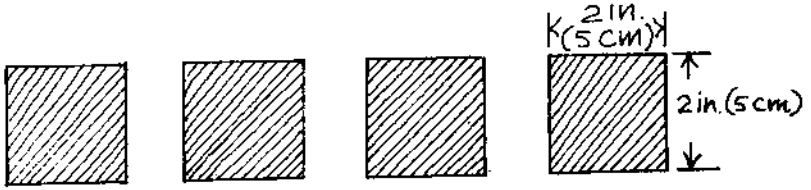
Exercises

1. A rectangle has measurements of 100 in. (254 cm) × 59 in. (150 cm). What is the perimeter of the structure?

2. Determine the perimeter of this irregularly shaped polygon.



3. Arrange the four squares to form a structure with a perimeter of:
   (a) 16 in. (40 cm)
   (b) 20 in. (50 cm)
   (c) 24 in. (60 cm)



Activity: ROLLER

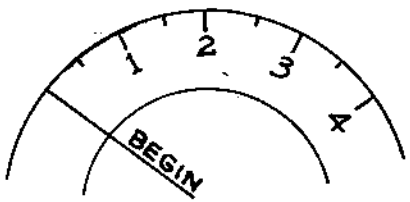
Purpose To construct and use a measuring wheel.

Materials

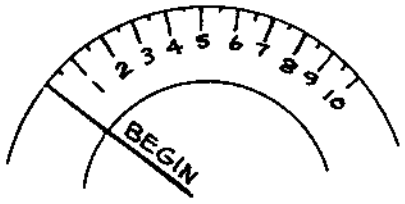
- scissors
- index card
- pencil
- marking pen
- ruler
- book
- coffee can lid

Procedure Use these instructions to make a measuring wheel.

- Cut a ½ in. × 1 in. (1 cm × 3 cm) strip from the index card.
- Mark a line in the center of the shorter sides of the paper strip to indicate ¼ in. (½ cm) length.
- Use the marking pen to draw a 2 in. (5 cm) line from the lid's edge toward the center of the lid.
- With the marking pen, write the word BEGIN on the 2 in. (5 cm) line on the lid.

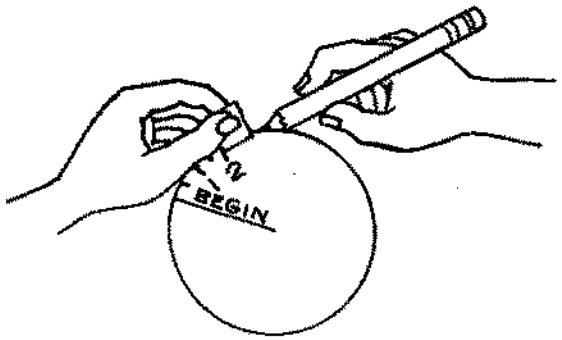


INCHES

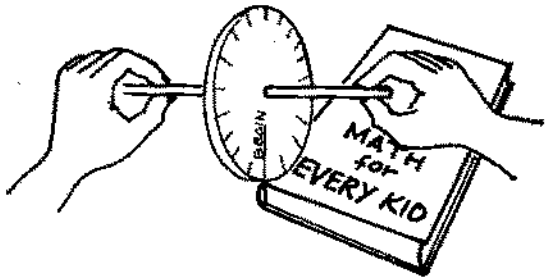


CENTIMETERS

- Use the paper strip to indicate the position of ¼ in. (½ cm) sections around the edge of the lid. Start with the BEGIN line and mark each ¼ in. (½ cm) section with the marking pen.



- To measure inches, number every fourth line. (Number every second line to measure centimeters.)
- Insert one-half the length of the pencil through the center of the plastic lid.
- Place the BEGIN line on the edge of a book.



- Measure the perimeter of the book by holding the pencil and rolling the lid around the outer edge of the book.

Results The perimeter of the book is determined by the number of turns of the lid plus any fraction of a turn.

Did You Know?

Trundle wheels similar to your measuring wheel are used to measure distances. The perimeter of houses or distances between the bases on a baseball field can be quickly measured with a trundle wheel that measures one meter with each turn.

Solutions

1. The perimeter of the rectangular shape can be determined by adding the length of all four sides together:

   100 in. + 59 in. + 100 in. + 59 in. = 318 in.
   (254 cm + 150 cm + 254 cm + 150 cm = 808 cm)

   or

   by determining the distance halfway around the rectangle and multiplying by two:

   English

   Step 1 100 in. + 59 in. = 159 in.
   Step 2 159 in. × 2 = 318 in.

   Metric

   Step 1 254 cm + 150 cm = 404 cm
   Step 2 404 cm × 2 = 808 cm

2. The perimeter of irregularly shaped polygons can be determined by adding the length of all the sides together.

   English

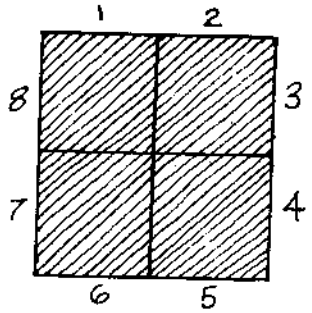
   2 in. + 4 in. + 5 in. + 8 in. + 6 in. = 25 in.

   Metric

   5 cm + 10 cm + 12.7 cm + 20.3 cm + 15 cm = 63 cm

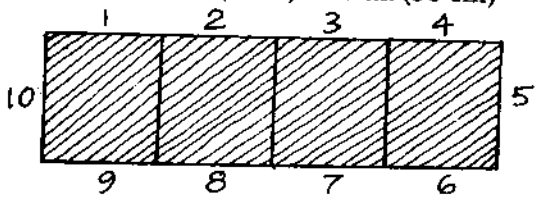
3 a. Arrange the four squares to form one large square.

Think! Number of sides × Length of each side = Perimeter
8 sides × 2 in. (5 cm) = 16 in. (40 cm)



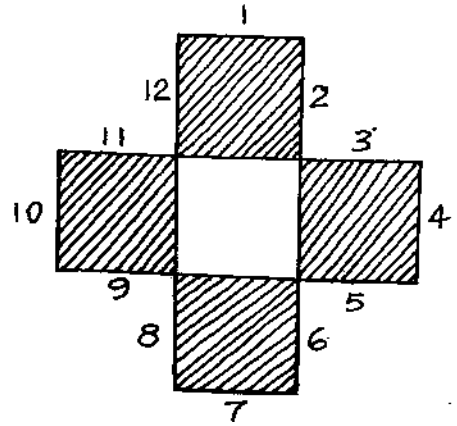
b. Place the squares side by side to form a straight line.

Think! Number of sides × Length of each side = Perimeter
10 sides × 2 in. (5 cm) = 20 in. (50 cm)



c. Arrange the squares to form a cross shape.

Think! Number of sides × Length of each side = Perimeter
12 sides × 2 in. (5 cm) = 24 in. (60 cm)

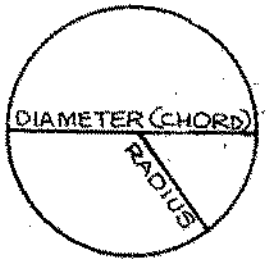


9

Diameter of a Circle

Purpose To measure the diameter of a circle.

Facts A straight line that begins and ends on a circle is called a **chord.** A chord that passes through the center of a circle is called a **diameter.** Any line that connects the center of a circle to any point on the circle's edge is called a **radius.** A radius is equal to one-half the length of the diameter.

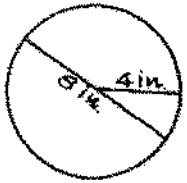


Problems

Question Study the diagrams and determine either the length of the radius or the diameter of each.

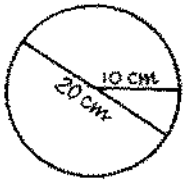
English

a. Diameter = 8 in.
Radius = ½ of the diameter
= ½ × 8 in.
= 4 in.



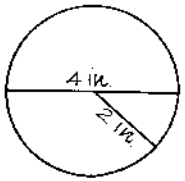
Metric

Diameter = 20 cm
Radius = ½ of the diameter
= ½ × 20 cm
= 10 cm



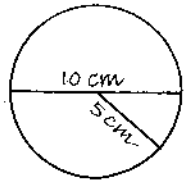
b. Radius = 2 in.
Diameter = 2 × radius
= 2 × 2 in.
= 4 in.

English



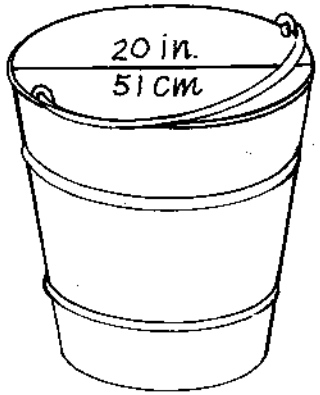
Metric

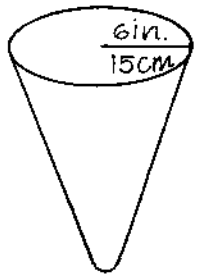
Radius = 5 cm
Diameter = 2 × radius
= 2 × 5 cm
= 10 cm



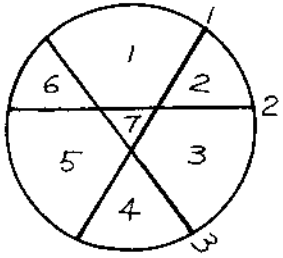
Exercises

1. Study the diagrams and determine the length of the radius and the diameter of each.





***2.** Three chords are used in the diagram to cut the circle into 7 parts. Use 5 chords to cut a circle into 16 parts.



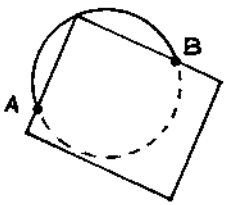
Activity: CENTER POINT

Purpose To find the center of a circle.

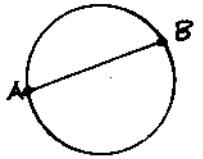
Materials
index card
pencil
paper
typing paper
drinking glass
ruler

Procedure

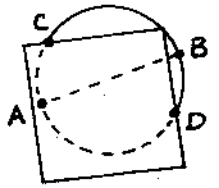
- Place the glass upside down on the paper.



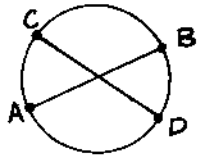
- Use the pencil to draw a circle by marking around the outside of the glass.
- Remove the glass from the paper.



- Place a corner of the index card so that it touches any point on the edge of the circle, then mark points A and B on the circle as in the diagram.
- Use the ruler to draw a line between points A and B.



- Place the corner of the card so that it touches a different place on the circle's edge, and mark points C and D as in the diagram.
- Use the ruler to draw a line between points C and D.



Results The lines intersect in the center of the circle. It will not matter where you place the card or how many lines you draw, they will all intersect in the center of the circle.

Did You Know?

The average diameter of a human hair is about 0.0635 mm.

Solutions

1. a. English

Radius = 4 in.
Diameter = 2 × radius
= 2 × 4 in.
= 8 in.

Metric

Radius = 10 cm
Diameter = 2 × radius
= 2 × 10 cm
= 20 cm

b. English

Diameter = 20 in.
Radius = ½ of diameter
= ½ × 20 in.
= 10 in.

Metric

Diameter = 51 cm
Radius = ½ diameter
= ½ × 51 cm
= 25.5 cm

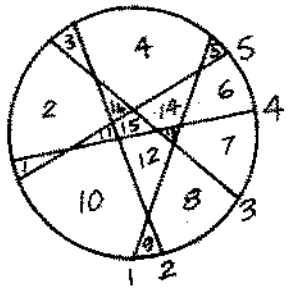
c. English

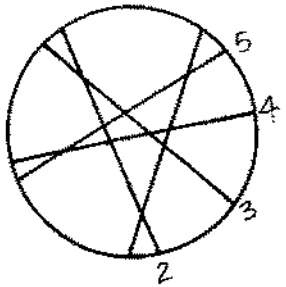
Radius = 6 in.
Diameter = 2 × radius
= 2 × 6 in.
= 12 in.

Metric

Radius = 15 cm
Diameter = 2 × radius
= 2 × 15 cm
= 30 cm

***2.**





10
Circumference of a Circle

Purpose To determine the circumference of a circle using the formula $c = \pi \times d$.

Facts The formula $c = \pi \times d$ is read as:

Circumference = pi times Diameter

The **circumference** of any circle divided by its diameter equals approximately 3.14. This number, 3.14, is called pi (symbolized by π, a Greek letter) and is the same for all circles regardless of their size.

Note: You have just learned one of the really neat things about math. There are *relationships* that, once discovered, hold true everywhere and for everyone in the universe! No matter how big the circle is or what it is made of or who makes it, pi is *always* 3.14.

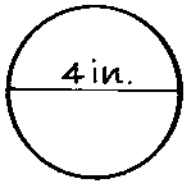
Problem

Question Determine the circumference of each circle using the formula.

1. a.

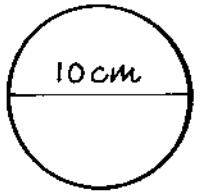
English

Diameter = 4 in.
π = 3.14
Formula: $c = \pi \times d$
$= 3.14 \times 4$ in.
$= 12.56$ in.



Metric

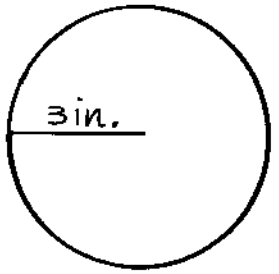
Diameter = 10 cm
π = 3.14
Formula: $c = \pi \times d$
$= 3.14 \times 10$ cm
$= 31.4$ cm



1. b.

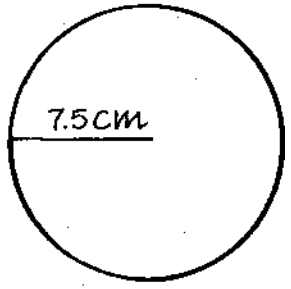
English

Radius = 3 in.
Diameter = 2×3 in. = 6 in.
π = 3.14
Formula: $c = \pi \times d$
$= 3.14 \times 6$ in.
$= 18.84$ in.



Metric

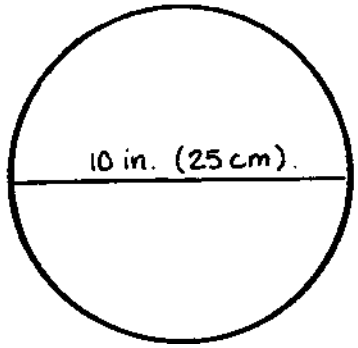
Radius = 7.5 cm
Diameter = 2×7.5 cm = 15 cm
π = 3.14
Formula: $c = \pi \times d$
$= 3.14 \times 15$ cm
$= 47.1$ cm



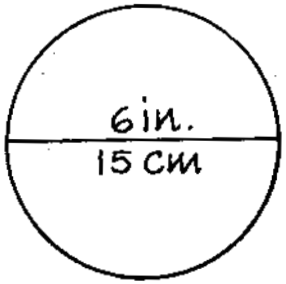
Exercises

1. Determine the circumference of a circle with

a. A diameter of 10 in. (25 cm)



b. A radius of 6 in. (15 cm)



2. A child is swinging an 80-in. (200-cm) long rope. How far does the ball attached to the end of the rope travel in one complete turn?

3. A phonograph record has a radius of 5.5 in. (14 cm). How far does a point on the edge of the record travel in four turns?

Activity: RUN AROUND

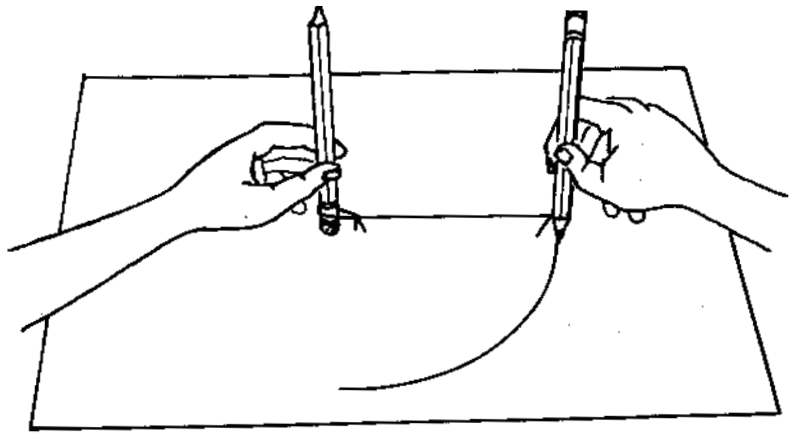
Purpose To draw circles of different diameters.

Materials

2 pencils	scissors
string	ruler
paper	

Procedure

- Cut a piece of string about 6 in. (15 cm) long.
- Tie one end of the string around a pencil and tie a loop in the other end of the string.
- Place the loop in the center of the paper.
- Stand a second pencil in the center of the loop with the eraser touching the paper and hold this pencil stationary.
- Pull the pencil tied to the string outward to stretch the string.



- Move the tied pencil around with its point pressed against the paper until a complete circle is drawn.
- Change the length of the string and repeat.

Results The point of the pencil marks the outline of a circle. The length of the string is equal to the **radius** of the circle. As the length of the string—the radius—increases, the size of the circle increases.

Did You Know?

The circumference of the earth at the equator is 24,901.46 miles (39,842.336 km). The circumference around the earth's poles is 45 miles (72 km) less than around the equator.

Solutions

1. a.

English

Diameter = 10 in.

$\pi = 3.14$

Formula: $c = \pi \times d$

$= 3.14 \times 10$ in.

$= 31.4$ in.

Metric

Diameter = 25 cm

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 25$

= 78.5 cm

1. b.

English

Radius = 6 in.

Diameter = 2×6 in. = 12 in.

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 12$ in.

= 37.68 in.

Metric

Radius = 15 cm

Diameter = 2×15 cm = 30 cm

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 30$ cm

= 94.2 cm

2. The length of the rope equals the radius of the circle, and the distance traveled by the ball in one turn equals the circumference of the circle.

English

Radius = 80 in.

Diameter = 2×80 in. = 160 in.

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 160$ in.

= 502.4 in.

Metric

Radius = 200 cm

Diameter = 2×200 cm = 400 cm

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 400$ cm

= 1256 cm

3. The distance traveled by a point on the edge of the record in four turns is equal to four times the circumference of the record.

English

Radius = 5.5 in.

Diameter = 2×5.5 in. = 11 in.

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 11$ in.

= 34.54 in.

Total distance = $4 \times c$

$= 4 \times 34.54$ in.

= 138.16 in.

Metric

Radius = 14 cm

Diameter = 2×14 cm = 28 cm

π = 3.14

Formula: $c = \pi \times d$

$= 3.14 \times 28$ cm

= 87.92 cm

Total distance = $4 \times c$

$= 4 \times 87.92$ cm

= 351.68 cm

11
Area of Rectangles and Squares

Purpose To find the area of a rectangle or square using the formula $A = L \times W$.

Facts The formula $A = L \times W$ is read as:

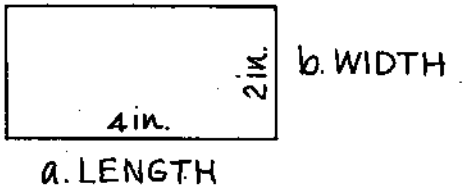
Area = Length times Width

Sides a and b of the diagram may be labeled as either the length or the width without changing the result.

Example 1

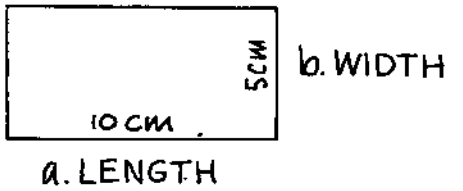
English

A = Length × Width
= 4 in. × 2 in.
= 8 in.^{2}



Metric

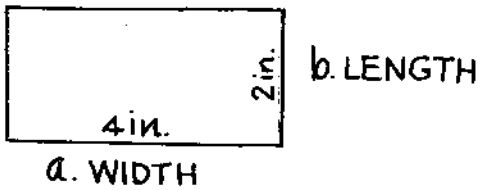
A = Length × Width
= 10 cm × 5 cm
= 50 cm^2



Example 2

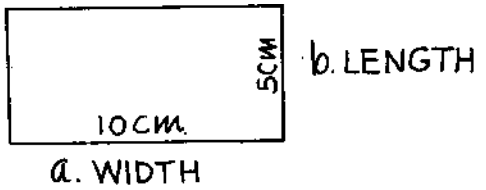
English

A = Length × Width
= 2 in × 4 in.
= 8 in.^{2}



Metric

A = Length × Width
= 10 cm × 5 cm
= 50 cm^2



When two units are multiplied, such as ft × ft, a small 2 is placed to the upper right of the unit, ft^2, and the combination is read as square feet. (m^2 is read as square meters.)

Problem

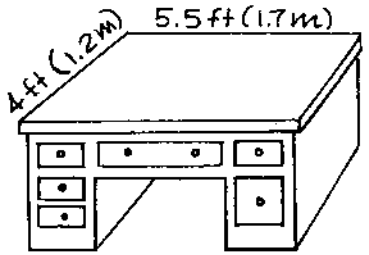
Question If the president's desk in the Oval Office is 5.5 ft (1.7 m) long and 4 ft (1.2 m) wide, what is the desk's surface area?

English

A = Length × Width
= 4 ft × 5.5 ft
= 22 ft^2

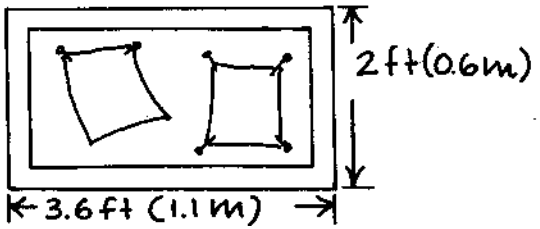
Metric

A = Length × Width
= 1.2 m × 1.7 m
= 2.04 m^2

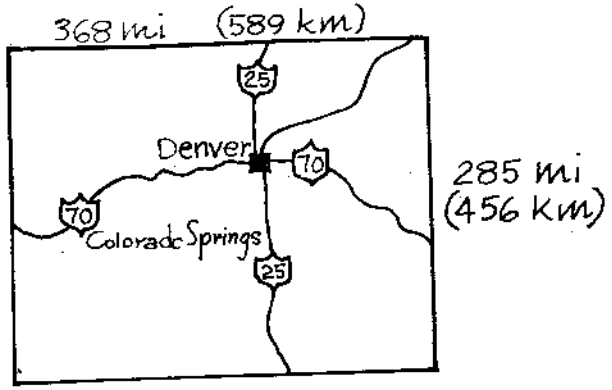


Exercises

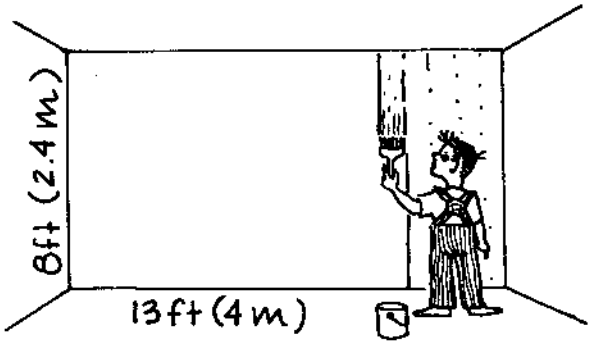
1. What is the area of the bulletin board?



2. Colorado is nearly rectangular. Determine its area.



***3.** One quart (liter) of paint will cover an area of 110 ft^2 (10.2 m^2). Is one quart (liter) enough paint to cover a wall 13 ft (4 m) wide and 8 ft (2.4 m) high?



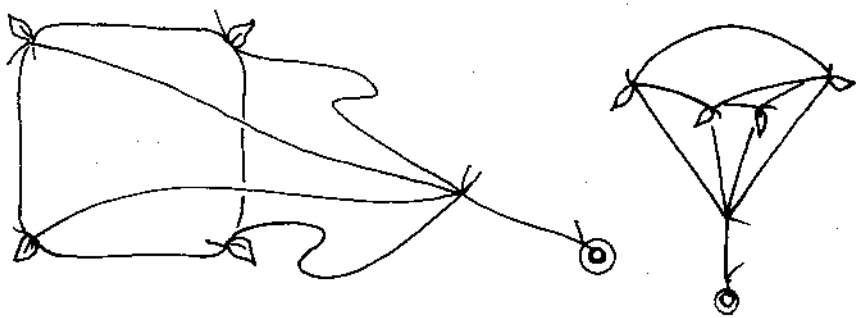
Activity: BIGGER

Purpose To determine how area affects the speed of falling objects.

Materials
- plastic garbage bag
- scissors
- string
- ruler
- 2 small washers of equal size and weight

Procedure

- Cut eight strings, each about 24 in. (60 cm) long.
- Measure and cut a 10 in. (25 cm) square from the plastic bag.
- Tie a string to each corner of the plastic sheet (parachute).
- Be sure the four free strings are the same length, then tie all four ends together in a knot.
- Use a string about 6 in. (15 cm) long to attach a washer to the knot joining the parachute strings.



- Make a second larger parachute using a 24 in. (60 cm) square of plastic and the four remaining strings.
- Tie the four strings in a knot and attach the washer to the knot with a 6 in. (15 cm) string as before.
- To test the parachutes, hold each in the center of the plastic sheet. Flatten the plastic.
- Fold the plastic in half.
- Loosely wrap the string around the folded plastic.
- Throw the parachutes up into the air, one at a time, and observe the time it takes for each to reach the ground.

Results The larger parachute opens and floats to the ground more slowly than does the smaller parachute. The washers are the same weight, with the same air resistance, and do not affect the relative speed.

Did You Know?

Objects hit against air as they fall. The larger the surface of a falling object, the more air it collects as it falls. Gravity pulls things down, but the collected air under the falling object pushes upward. A parachute with its collected air slows the fall of a person. Some insects have such a large surface area as compared to their weight that they can fall from a tall building and walk away unharmed.

Solutions

1.

English

A = Length × Width
= 3.6 ft^2 × 2 ft
= 7.2 ft^2

Metric

A = Length × Width
= 1.1 m × .6 m
= .66 m^2

2.

English

A = Length × Width
= 368 mi × 285 mi
= 104,880 mi^2

Metric

A = Length × Width
A = 589 km × 456 km
= 268,584 km^2

3.

English

A = Length × Width
= 13 ft × 8 ft
= 104 ft^2

Think! 110 ft^2 is covered by 1 quart. 104 ft^2 is less than 110 ft^2.

Answer Yes, 1 quart is enough paint.

Metric

A = Length × Width
= 4 m × 2.4 m
= 9.6 m^2

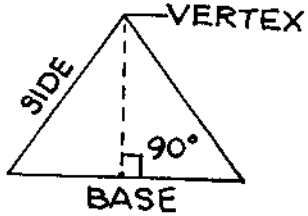
Think! 10.2 m^2 is covered by 1 quart. 9.6 m^2 is less than 10.2 m^2.

Answer Yes, 1 liter is enough paint.

12
Area of Triangles

Purpose To find the area of a triangle using the formula $A = \frac{1}{2} \times b \times h$.

Facts **Triangle:** a plane with three sides that meet to form three vertices or points. **Plane:** Any flat surface. **Vertices:** Plural for vertex. The vertex is the point formed when two straight lines meet at an angle. **Perpendicular:** Two lines that form a 90° (90 degree) angle.



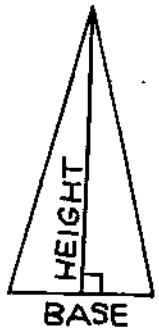
The formula

$$A = \frac{1}{2} \times b \times h$$

is read as:

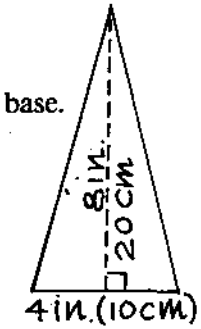
Area = One-half times Base times Height

The height of a triangle is a straight perpendicular line from a side to a vertex. The side is called the base and a small box is drawn between the height line and the base to indicate that they meet at a 90° angle.



Problems

Question Find the area of the triangle.



The height is the line that forms a 90° line with the base.

English
Formula: $A = \frac{1}{2} \times b \times h$
Height = 8 in.
Base = 4 in.
Area = $\frac{1}{2} \times 4$ in. $\times$ 8 in.

When multiplying three numbers together, work with two numbers at a time. Multiply the first two and then multiply the product of these two numbers by the third number.

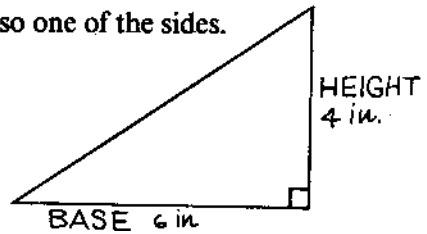
Think! $\frac{1}{2} \times 4$ in. = 2 in.
thus
$A = 2 \text{ in.} \times 8 \text{ in.} = 16 \text{ in.}^2$

Metric
Formula: $A = \frac{1}{2} \times b \times h$
Height = 20 cm
Base = 10 cm
Area = $\frac{1}{2} \times 10 \text{ cm} \times 20 \text{ cm}$

Think! $\frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$
thus
$A = 5 \text{ cm} \times 20 \text{ cm} = 100 \text{ cm}^2$

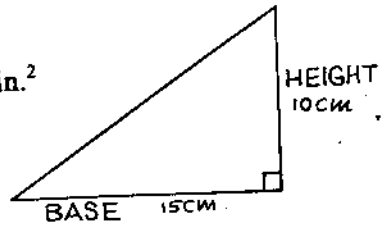
Question Find the area of the triangle.

The height line in this triangle is also one of the sides.



English
Formula: $A = \frac{1}{2} \times b \times h$
Height = 4 in.
Base = 6 in.
Area = $\frac{1}{2} \times 6$ in. $\times$ 4 in.

Think! $\frac{1}{2} \times 6$ in. = 3 in.
thus
$A = 3 \text{ in.} \times 4 \text{ in} = 12 \text{ in.}^2$



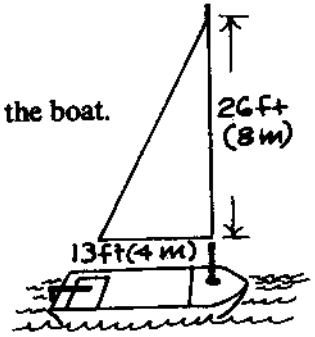
Metric

Formula: $A = \frac{1}{2} \times b \times h$
Height = 10 cm
Base = 15 cm
Area = $\frac{1}{2} \times 15 \text{ cm} \times 10 \text{ cm}$

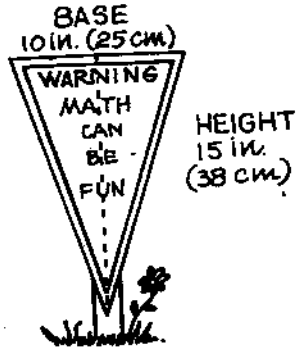
Think! $\frac{1}{2} \times 15 \text{ cm} = 7.5 \text{ cm}$
thus
$A = 7.5 \text{ cm} \times 10 \text{ cm} = 75 \text{ cm}^2$

Exercises

1. Find the area of the sail on the boat.



2. Find the area of the sign if it has a height of 15 in. (38 cm) and a base of 10 in. (25 cm).



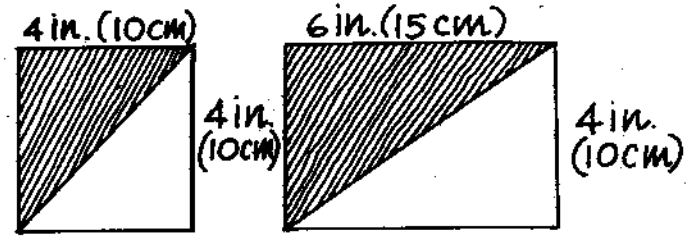
Activity: EQUALS

Purpose To demonstrate how the area formula for triangles, $A = \frac{1}{2} \times b \times h$, is determined.

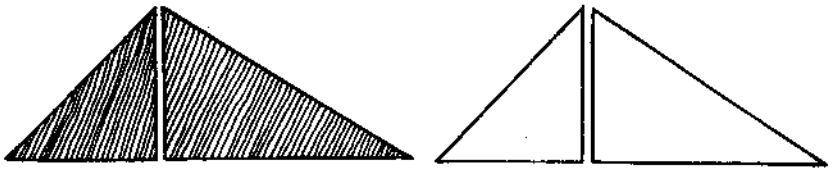
Materials
pencil
ruler
scissors
red crayon
typing paper

Procedure

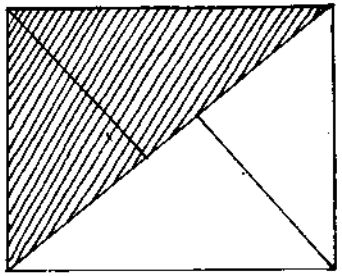
- Use the pencil to draw two shapes, a 4 in. (10 cm) × 6 in. (15 cm) rectangle, and a 4 in. (10 cm) square.



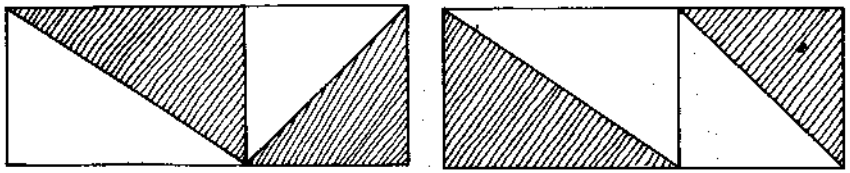
- Draw a diagonal line across each of the figures.
- Color one triangle in each figure red, leaving the remaining two triangles uncolored.
- Use the scissors to cut out the four triangles.
- Arrange the four pieces to form two separate triangles, one colored and one uncolored.



- Compare the sizes of the two triangles.
- Combine the two triangles to form one rectangle.



- Rearrange the four pieces to change the size of the rectangle.



Results The rectangle is made of two triangles, each with the same total surface area. The area of a rectangle is calculated by using the formula

$$A = \text{Length (Base)} \times \text{Width (Height)}.$$

Since each triangle is one-half the area of the rectangle, the area of each separate triangle can be calculated by multiplying the area of the rectangle by 1/2. A formula for the area of each triangle would be

$$A = 1/2 \times \text{Base} \times \text{Height}.$$

Did You Know?

The largest pyramid, Quetzalcoatl, is located in Mexico. This structure is 177 ft (54.5 m) tall with a base area of 45 acres.

Solutions

1.

English

Formula: $A = 1/2 \times b \times h$
Height = 26 ft
Base = 13 ft
Area = $1/2 \times 13\text{ ft} \times 26\text{ ft}$

Think! $1/2 \times 13\text{ ft} = 6.5\text{ ft}$
thus
$A = 6.5\text{ ft} \times 26\text{ ft} = 169\text{ ft}^2$

Metric

Formula: $A = 1/2 \times b \times h$
Height = 8 m
Base = 4 m
Area = $1/2 \times 4\text{ m} \times 8\text{ m}$

Think! $1/2 \times 4\text{ cm} = 2\text{ m}$
thus
$A = 2\text{ m} \times 8\text{ m} = 16\text{ m}^2$

2.

English

Formula: $A = 1/2 \times b \times h$
Height = 15 in.
Base = 10 in.
Area = $1/2 \times 10\text{ in.} \times 15\text{ in.}$

Think! $1/2 \times 10\text{ in.} = 5\text{ in.}$
thus
$A = 5\text{ in.} \times 15\text{ in.} = 75\text{ in}^2$

Metric

Formula: $A = 1/2 \times b \times h$
Height = 38 cm
Base = 25 cm
Area = $1/2 \times 25\text{ cm} \times 38\text{ cm}$

Think! $1/2 \times 25\text{ cm} = 12.5\text{ cm}$
thus
$A = 12.5\text{ cm} \times 38\text{ cm} = 475\text{ cm}^2$

13
Area of Circles

Purpose To find the area of a circle using the formula $A = \pi r^2$.

Facts The formula $A = \pi r^2$ is read as:

Area = pi times radius times radius
or
pi times radius squared

Since pi (π) is always the same number—3.14—the formula can be written as:

$$\text{Area} = 3.14 \times \text{Radius} \times \text{Radius} = 3.14r^2$$

Problems

Question A circular rug has a radius of 7 ft (2 m). What is the surface area of the rug?

English

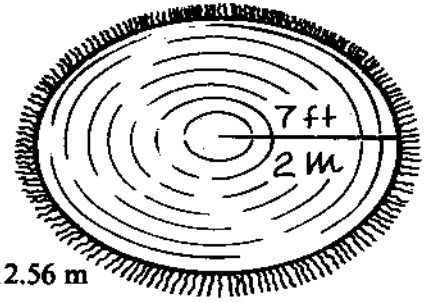
Formula: $A = \pi \times r \times r$
Radius = 7 ft
$= 3.14$
$A = 3.14 \times 7\text{ ft} \times 7\text{ ft}$

Think! $3.14 \times 7\text{ ft} = 21.98\text{ ft}$
thus
$A = 21.98\text{ ft} \times 7\text{ ft} = 153.86\text{ ft}$

When multiplying three numbers together, work with two numbers at a time. Multiply the first two, then multiply the product of these two numbers by the third number.

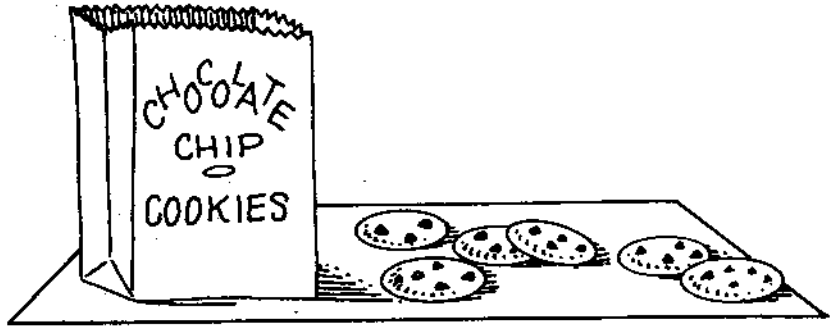
Metric

Formula: $A = \pi \times r \times r$
Radius = 2 m
$= 3.14$
$A = 3.14 \times 2\text{ m} \times 2\text{ m}$



Think! $3.14 \times 2\text{ m} = 6.28\text{ m}$
thus
$A = 6.28\text{ m} \times 2\text{ m} = 12.56\text{ m}$

Question Find the area of a cookie that has a diameter of 4 in. (10 cm).



English

Formula: $A = \pi \times r \times r$
Diameter = 4 in.
Radius = $\frac{1}{2} \times$ diameter
$= \frac{1}{2} \times 4\text{ in.} = 2\text{ in.}$
$\pi = 3.14$
$A = 3.14 \times 2\text{ in.} \times 2\text{ in.}$

Think! $3.14 \times 2\text{ in.} = 6.28\text{ in.}$
thus
$A = 6.28\text{ in.} \times 2\text{ in.} = 12.56\text{ in.}^2$

Metric

Formula: $A = \pi \times r \times r$

Diameter = 10 cm

Radius = $\frac{1}{2}$ × diameter

$= \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$

$\pi = 3.14$

$A = 3.14 \times 5 \text{ cm} \times 5 \text{ cm}$

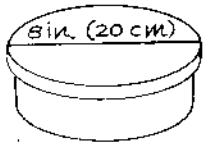
Think! $3.14 \times 5 \text{ cm} = 15.7 \text{ cm}$

thus

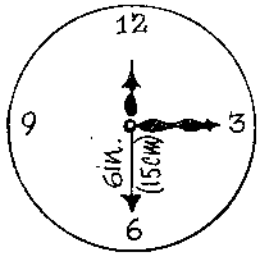
$A = 15.7 \text{ cm} \times 5 \text{ cm} = 78.5 \text{ cm}^2$

Exercises

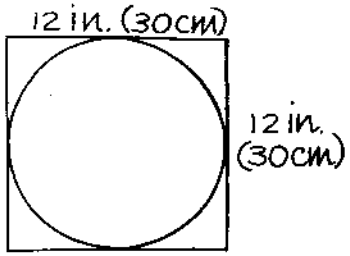
1. Determine the area of the lid on the can.



2. The second hand of the clock is 6 in. (15 cm) long. Determine the area that the hand sweeps in 1 minute.



***3** A circle was cut from a 12 in. (30 cm) square of material. How much material was not used?



Activity: HOW BIG?

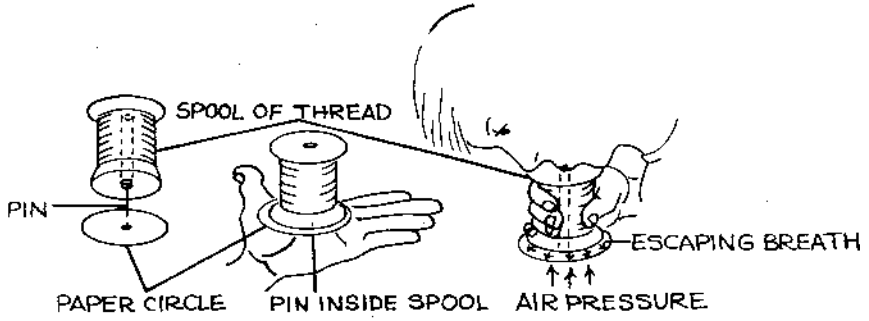
Purpose To demonstrate the effect a change in radius has on the size of a circle.

Materials 3 circular objects with diameters of approximately 2 in. (5 cm), 4 in. (10 cm), and 6 in. (15 cm) (*Note:* The exact size is not important.)

spool of thread	ruler
pencil	scissors
typing paper	straight pin

Procedure

- Use the circular objects to draw three separate circles on the typing paper with diameters of approximately 2 in. (5 cm), 4 in. (10 cm), and 6 in. (15 cm).
- Use scissors to cut each circle from the paper.
- Stick a straight pin through the center of the smallest circle.
- Place the paper in the palm of your hand with the pin pointing up.
- Remove any paper covering the hole through the thread spool, then position the hole in the spool over the pin.
- While holding the spool, blow into the top of the hole.
- Remove your hand from beneath the paper while continuing to blow through the spool.
- Repeat the procedure using the larger paper circles.



Results The smaller circles do not fall, but stay on the bottom of the spool. The air flows out between the paper and the spool, producing a low-pressure area. The air beneath the paper pushes up with enough force to keep the paper from falling. The circles finally get too large for the air to hold up. The largest circle falls.

Did You Know?

The largest recorded pizza had a diameter of 1,201 in. (3051 cm). It was cut into 94,248 pieces.

Solutions

1.

English

Formula: $= \pi \times r \times r$
Diameter $= 8$ in.
Radius $= \frac{1}{2} \times$ diameter
$= \frac{1}{2} \times 8$ in. $= 4$ in.
$\pi = 3.14$
$A = 3.14 \times 4$ in. $\times 4$ in.

Think! 3.14×4 in. $= 12.56$ in.
thus
$A = 12.56$ in. $\times 4$ in. $= 50.24$ in.^{2}

Metric

Formula: $= \pi \times r \times r$
Diameter $= 20$ cm
Radius $= \frac{1}{2} \times$ diameter
$= \frac{1}{2} \times 20$ cm $= 10$ cm
$\pi = 3.14$
$A = 3.14 \times 10$ cm $\times 10$ cm

Think! 3.14×10 cm $= 31.4$ cm
thus
$A = 31.4$ cm $\times 10$ cm $= 314$ cm^2

2.

English

Formula: $A = \pi \times r \times r$
Radius $= 6$ in.
$A = 3.14 \times 6$ in. $\times 6$ in.

Think! 3.14×6 in. $= 18.84$ in.
thus
$A = 18.84$ in. $\times 6$ in. $= 113.04$ in.^{2}

Metric

Formula: $A = \pi \times r \times r$
Radius $= 15$ cm
$A = 3.14 \times 15$ cm $\times 15$ cm

Think! 3.14×15 cm $= 47.1$ cm
thus
$A = 47.1$ cm $\times 15$ cm $= 706.5$ cm^2

***3.** Calculate the area of the 30-cm square of material and subtract the area of the circle to determine the amount of unused material.

English

Formula: $A =$ Length $\times$ Width
Length $= 12$ in.
Width $= 12$ in.
$A = 12$ in. $\times 12$ in.
$= 144$ in.^{2}

Metric

Formula: $A =$ Length $\times$ Width
Length $= 30$ cm
Width $= 30$ cm
$A = 30$ cm $\times 30$ cm
$= 900$ cm^2

English

Formula: $A = \pi \times r \times r$

Diameter = 12 in.

Radius = $1/2 \times$ Diameter

$= 1/2 \times 12$ in. $= 6$ in.

$\pi = 3.14$

$A = 3.14 \times 6$ in. $\times 6$ in.

Think! 3.14×6 in. $= 18.84$ in.

thus

$A = 18.84$ in. $\times 6$ in. $= 113.04$ in.^{2}

Area of square	=	144.00 in.^{2}
− Area of circle	=	− 113.04 in.^{2}
Unused material	=	30.96 in.^{2}

Metric

Formula: $A = \pi \times r \times r$

Diameter = 30 cm

Radius = $1/2 \times$ Diameter

$= 1/2 \times 30$ cm $= 15$ cm

$\pi = 3.14$

$A = 3.14 \times 15$ cm $\times 15$ cm

Think! 3.14×15 cm $= 47.1$ cm

thus

$A = 47.1$ cm $\times 15$ cm $= 706.5$ cm^2

Area of square	=	900.0 cm
− Area of circle	=	− 706.5 cm
Unused material	=	193.5 cm^2

14
Surface Area

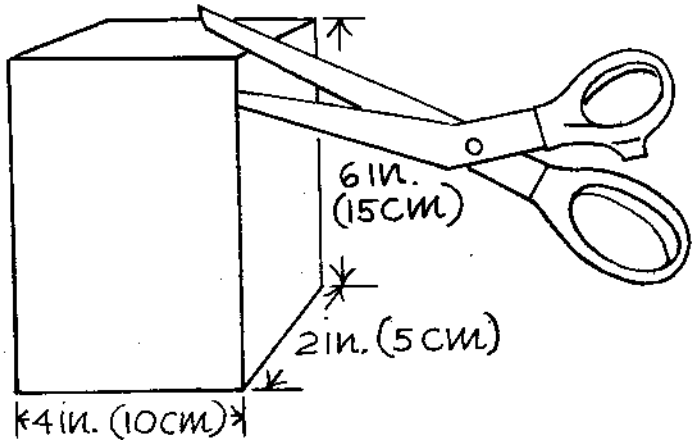
Purpose To determine the surface area of objects with different shapes.

Facts The **surface area** equals the total outside area of an object. The total surface area is the sum of the areas of the box's top, bottom, and four sides. Each of the six parts has the shape of a rectangle, so the area of each part can be determined by using the formula

Area = Length × Width.

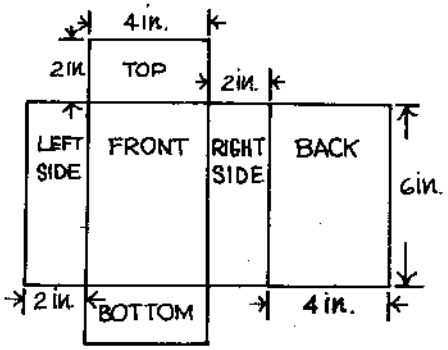
Problem

Question Determine the surface area of this closed box.



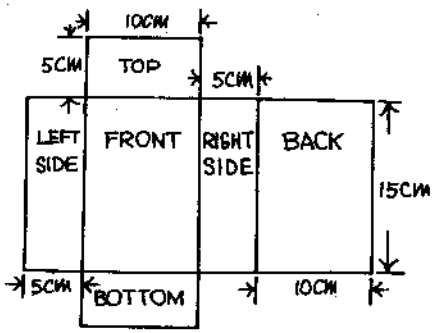
English

Top area	= 4 in. × 2 in.	=	8 in.^{2}
Bottom area	= 4 in. × 2 in.	=	8 in.^{2}
Left-side area	= 2 in. × 6 in.	=	12 in.^{2}
Front area	= 4 in. × 6 in.	=	24 in.^{2}
Right-side area	= 2 in. × 6 in.	=	12 in.^{2}
Back area	= 4 in. × 6 in.	=	24 in.^{2}
Surface area	=		88 in.^{2}



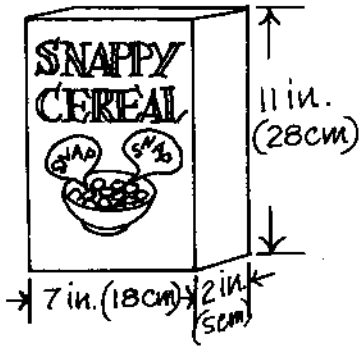
Metric

Top area	= 10 cm × 5 cm	=	50 cm^2
Bottom area	= 10 cm × 5 cm	=	50 cm^2
Left-side area	= 5 cm × 15 cm	=	75 cm^2
Front area	= 15 cm × 10 cm	=	150 cm^2
Right-side area	= 5 cm × 15 cm	=	75 cm^2
Back area	= 15 cm × 10 cm	=	150 cm^2
Surface area	=		262 cm^2

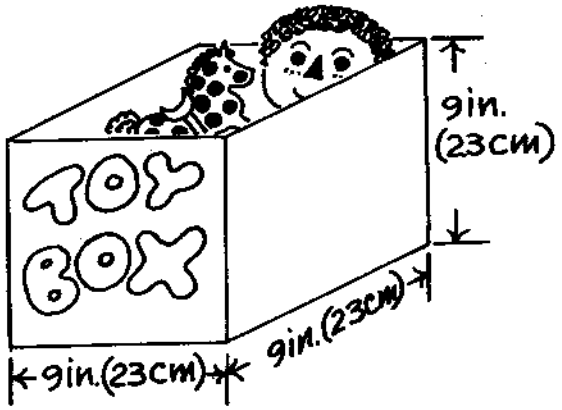


Exercises

1. Determine the surface area of the cereal box.



2. Determine the surface area of the open toy box.



Activity: RAINBOW NECKLACE

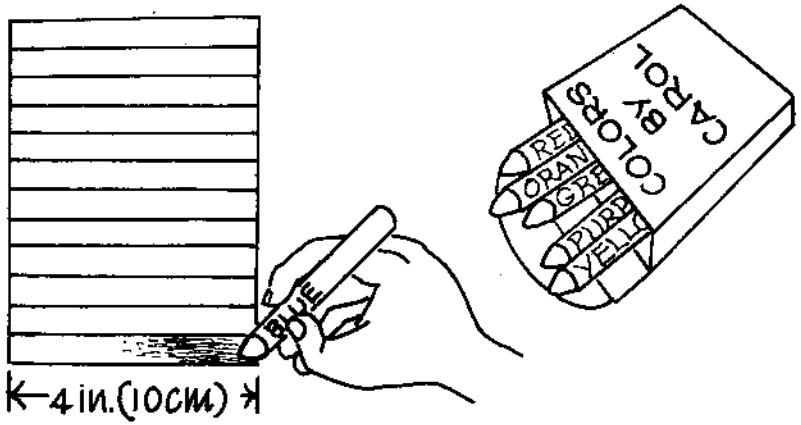
Purpose To demonstrate that surface area can remain the same even though the shape of an object changes.

Materials

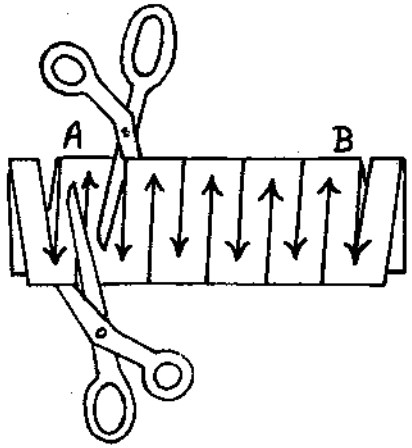
notebook paper	pencil
scissors	crayons
ruler	

Procedure

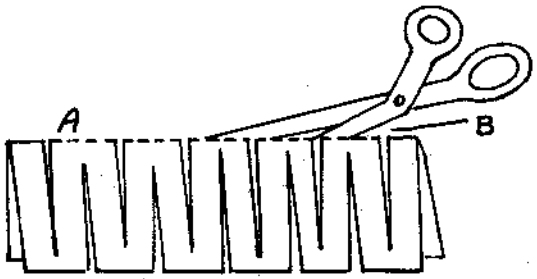
- Use a pencil to draw a rectangle on the lined paper that is 4 in. (10 cm) wide and 12 lines long.
- Cut the rectangle from the paper with the scissors.
- Use the crayons to color each of the 12 lines on the rectangle different colors.



- Fold the rectangle in half along the long side.
- Cut across the fold at points A and B. Stop about ¼ in. (1 cm) from the edge of the paper.



- Notice that all the cuts are along the lines printed on the paper and that each cut stops ¼ in. (1 cm) from the edge. The cuts alternate from the folded edge to the open edge.
- Cut along each of the printed lines alternating from the folded edge to the open edge. Be sure to stop ¼ in. (1 cm) from the edge.
- Start at point A and cut the folded edge off of the paper ending at point B. (*Note:* Do not cut the folded edge from the two end sections.)



- Carefully stretch the paper open and slip the rainbow-colored necklace around your neck.

Results The shape of the paper changed from a rectangle to a zig-zag chainlike structure, but the surface area of the paper remained the same.

Did You Know?

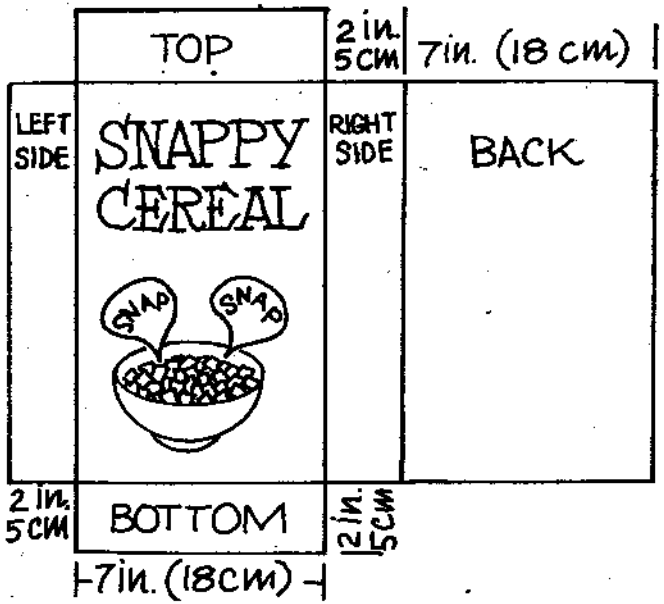
The small intestine of a human fits snugly inside the abdominal cavity. If this coiled tube is stretched out, its length is about 29 ft (9 m).

Solutions

1.

English

Top area	=	7 in. × 2 in.	= 14 in.^{2}
Bottom area	=	7 in. × 2 in.	= 14 in.^{2}
Left-side area	=	2 in. × 11 in.	= 22 in.^{2}
Front area	=	7 in. × 11 in.	= 77 in.^{2}
Right-side area	=	2 in. × 11 in.	= 22 in.^{2}
Back area	=	+ 7 in. × 11 in.	= 77 in.^{2}
Surface area	=		212 in.^{2}



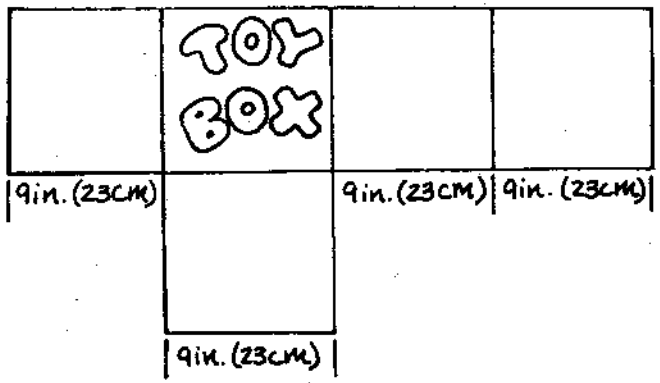
Metric

Top area	=	18 cm × 5 cm	= 90 cm^2
Bottom area	=	18 cm × 5 cm	= 90 cm^2
Left-side area	=	5 cm × 28 cm	= 140 cm^2
Front area	=	18 cm × 28 cm	= 504 cm^2
Right-side area	=	5 cm × 28 cm	= 140 cm^2
Back area	=	+ 18 cm × 28 cm	= 504 cm^2
Surface area	=		1378 cm^2

2. The toy box is an open box with five square sides. Since each of the five sides has the same area, the total surface area of the box can be determined by multiplying the area of one of the sides by five.

English

Area of one side = 9 in. × 9 in. = 81 in.^{2}
Total surface area = 81 in.^{2} × 5 = 405 in.^{2}



Metric

Area of one side = 23 cm × 23 cm = 529 cm^2
Total surface area = 529 cm^2 × 5 = 2645 cm^2

15
Volume of Cubes and Rectangular Prisms

Purpose To find the volume of cubes and rectangular prisms using the formula:

Volume = Length × Width × Height

The formula is abbreviated

$V = l \times w \times h$

Facts

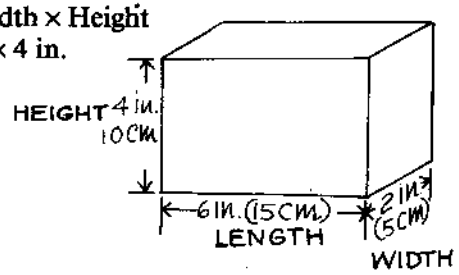
Cubes and rectangular prisms have three different measurements, length, width, and height. Changing the position of the box does not affect its volume but can change the labeling of the length, width, and height. When three units (such as in. × in. × in.) are multiplied, a small 3 is placed to the upper right of the unit ($in.^3$) and the combination is read as inches cubed or cubic inches.

Problems

Question Find the volume of the box.

English

Volume = Length × Width × Height
= 6 in. × 2 in. × 4 in.



When multiplying three numbers together, work with two numbers at a time. Multiply the first two and then multiply the product of these two numbers by the third number.

Think! 6 in. × 2 in. = $12\ in.^2$

then

Volume = $12\ in.^2 \times 4$ in. = $48\ in.^3$

Metric

Volume = Length × Width × Height
= 15 cm × 5 cm × 10 cm

Think! 15 cm × 5 cm = $75\ cm^2$

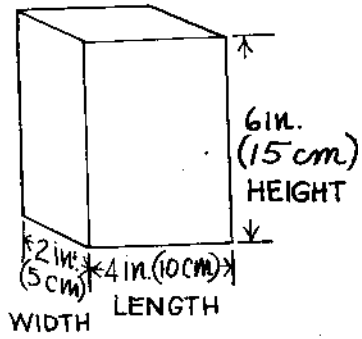
then

Volume = $75\ cm^2 \times 10$ cm = $750\ cm^3$

Question What is the volume of the box in Problem 1, if it is turned on its end?

English

Volume = Length × Width × Height
= 4 in. × 2 in. × 6 in.



Think! 4 in. × 2 in. = $8\ in.^2$

then

Volume = $8\ in.^2 \times 6$ in. = $48\ in.^3$

Metric

Volume = Length × Width × Height
= 10 cm × 5 cm × 15 cm

Think! 10 cm × 5 cm = $50\ cm^2$

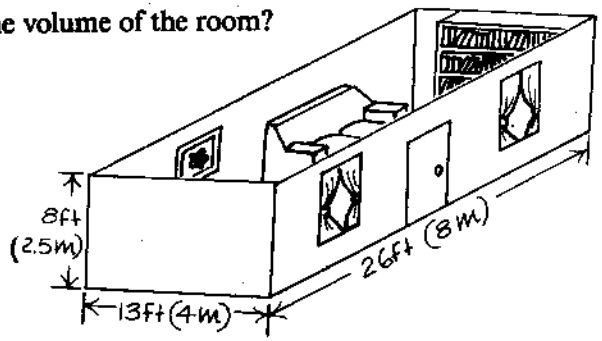
then

Volume = $50\ cm^2 \times 15$ cm = $750\ cm^3$

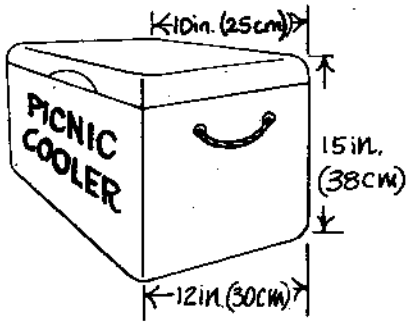
Note: Changing the position of box did not change the three measurements that were multiplied together, and the order in which the numbers are multiplied does not change the product.

Exercises

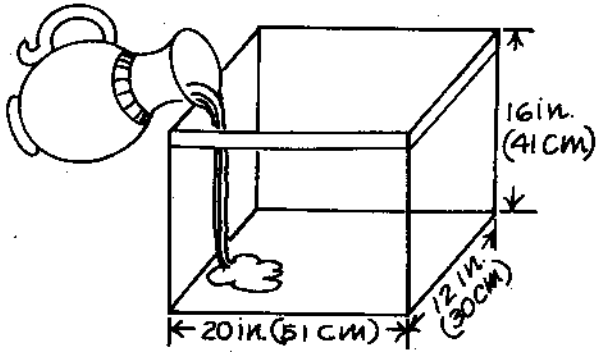
1. What is the volume of the room?



2. Calculate the volume of the picnic cooler.



***3.** A pitcher holding 122 in.^{3} (2000 cm^3) of water is used to fill an aquarium. Would 25 pitchers of water fill the aquarium?



Activity: MEASURING BOX

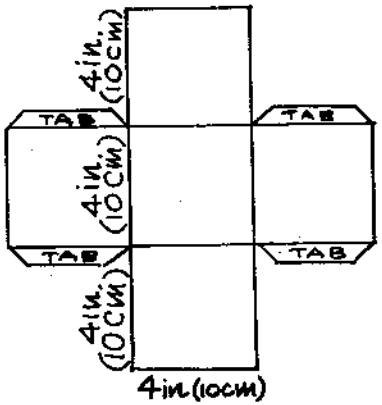
Purpose To determine how much water a 4 in. (10 cm) cube will hold.

Materials

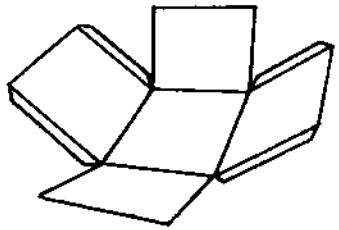
pencil
ruler
stiff paper, such as a file folder
1 qt (1 liter) soda bottle, empty
bowl large enough to hold the box
glue, white, all purpose
scissors

Procedure

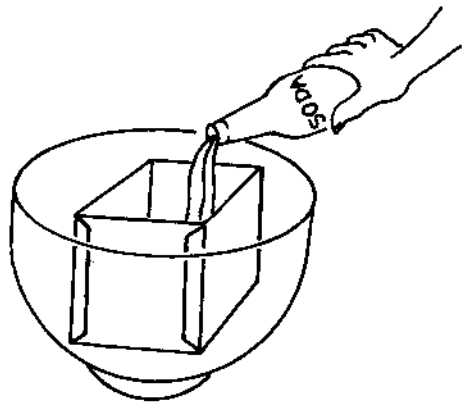
- On the stiff paper, draw a full-sized enlargement of the pattern shown using your ruler.



- Cut out the drawing and fold to make a cube with 4 in. (10 cm) sides.



- Use the glue to secure the tabs.
- Cover the seams inside the box with a generous layer of glue to make the box leakproof.
- Allow the glue to thoroughly dry.
- Fill the 1 qt (1 liter) bottle with water.
- Place the box in a bowl to catch any water spills.



- Slowly pour the water from the bottle into the box until the box is overflowing.

Results The box will hold 1 qt (1 liter) of water. A 4 in. (10 cm) cube has a volume of 64 in.^{3} (1000 cm^3). This volume equals 1 qt (1 liter).

Did You Know?

The largest box of popcorn measured 25 ft (7.7 m) x 25 ft (7.7 m) x 6.06 ft (1.86 m). The box was filled by Jones High School, Orlando, Florida, Dec 15–17, 1988.

Solutions

1.

English

Volume = Length × Width × Height
= 13 ft × 26 ft × 8 ft

Think! 13 ft × 26 ft = 338 ft^2
then
Volume = 338 ft^2 × 8 ft = 2704 ft^3

Metric

Volume = Length × Width × Height
= 4 m × 8 m × 2.5 m

Think! 4 m × 8 m = 32 m^2
then
Volume = 32 m^2 × 2.5 m = 80 m^3

2.

English

Volume = Length × Width × Height
= 12 in. × 10 in. × 15 in.

Think! 12 in. × 10 in. = 120 in.^{2}
then
Volume = 120 in.^{2} × 15 in. = 1800 in.^{3}

Metric

Volume = Length × Width × Height
= 30 cm × 25 cm × 38 cm

Think! 30 cm × 25 cm = 750 cm^2
then
Volume = 750 cm^2 × 38 cm = 28,500 cm^3

3.

English

1 pitcher = 122 in.^{3}
25 pitchers = 25 × 122 in.^{3}
= 3,050 in.^{3}

3,050 in.^{3} is less than the 3,840 in.^{3} volume of the aquarium.

Metric

1 pitcher = 2,000 cm^3
25 pitchers = 25 × 2,000 cm^3
= 50,000 cm^3

50,000 cm^3 is less than the 62,730 cm^3 volume of the aquarium.

Answer No, 25 pitchers of water will not fill the aquarium.

16
Volume by Displacement

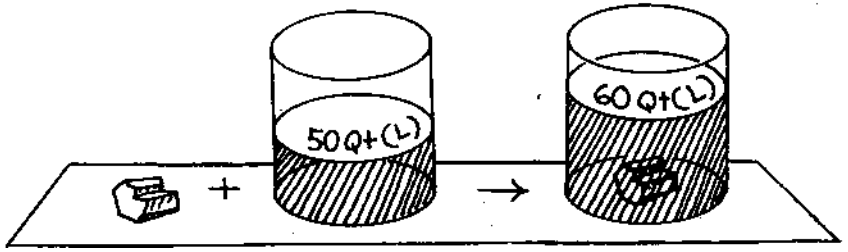
Purpose To calculate the volume of an object by water displacement.

Facts When an object is placed into a container of water, the amount of water the object pushes out of its way is equal to its **volume**. The water is being **displaced**, pushed aside, by the object; the object's volume is thus determined by the amount of water it displaces.

Problem

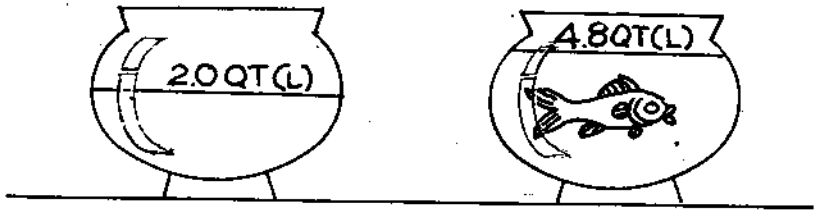
Question A rock is placed in 50 qt (50 liters) of water. The water level rises to 60 qt. (60 liters). What is the volume of the rock?

Water volume + Rock volume	=	60 qt (liters)
− Water volume	=	− 50 qt (liters)
Rock volume	=	10 qt (liters)

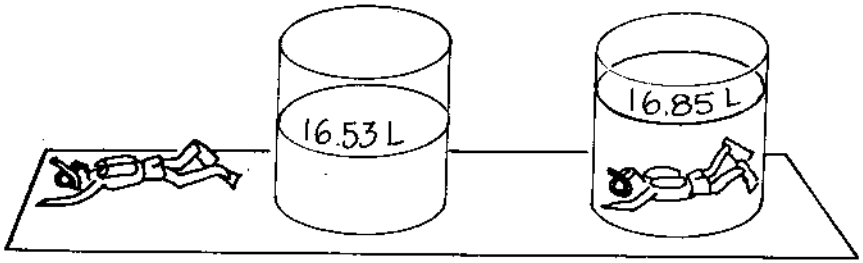


Exercises

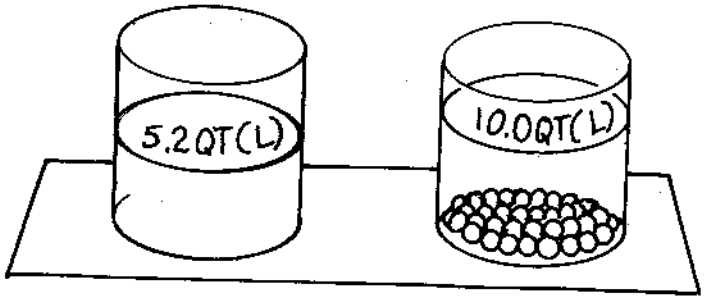
1. What is the volume of the fish?



2. How much of the water is displaced by the toy diver?



3. Each metal ball displaces 0.1 qt (0.1 liter). Study the picture to determine the number of balls in the jar.



Activity: SAME SIZE?

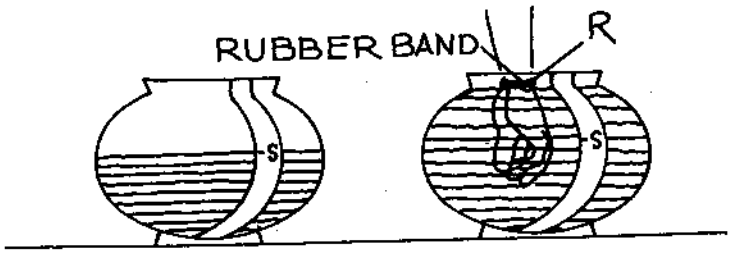
Purpose To compare the volume of your hands.

Materials

- 2 rubber bands
- fish bowl
- masking tape
- marking pen
- helper

Procedure

- Place a strip of tape down the side of the fish bowl.
- Fill the bowl three-quarters full with water.
- Use the pen to mark the water level on the tape. Label this starting mark with the letter S.
- Slip one rubber band around each of your wrists at exactly the same place. (*Note:* Be sure the rubber bands *do not* cut off your circulation. They should not be so tight as to cause the skin to pucker.)
- Push your left hand into the water until the rubber band is touching the surface of the water.
- Ask your helper to mark the level of the water on the tape and label the mark with the letter L.
- Remove your hand and add water to bring the level back to the starting mark, S.
- Push your right hand into the water until the rubber band touches the surface of the water.



■ Have your helper mark the level of the line on the tape with the letter R.

Results The lines R and L are very close together or possibly the letters are at the same position. Each hand pushes the water out of its way as it enters the liquid. The volume of each hand is equal to the amount of water it displaces. Your hands are not exactly the same, but the measuring instrument in this experiment is not sensitive enough to indicate small differences, so equal volumes for your hands may be shown. Volumes of objects such as hands, rocks, pieces of gold, or any irregular-shaped objects can be determined by water displacement.

> ***Did You Know?***
>
> ***The weight of the water that your hands displaced is about the same as the weight of your hands. This is because the weight of the human body is about the same as an equal volume of water.***

Solutions

1.

Water volume + Fish	=	4.8 qt (liters)
− Water volume	=	− 2.0 qt (liters)
Fish volume	=	2.8 qt (liters)

2.

Water volume + Volume of toy diver	=	16.85 qt (liters)
− Water volume	=	16.53 qt (liters)
Volume of diver	=	0.32 qt (liters)

3.

Water volume + Balls	=	10.0 qt (liters)
− Water volume	=	− 5.2 qt (liters)
Volume of balls	=	4.8 qt (liters)

To determine the number of balls, divide the displaced volume of all the balls by the volume of 1 ball.

4.8 qt (liters) ÷ 0.1 qt (liter) = 48 balls

17
Liquid Capacity

Purpose To measure and determine equivalent liquid capacities.

Facts

Unit	Abbreviation	Equivalent Measurements
quart	qt	1 liter = 1 qt
liter	L	1 liter = 1000 ml
milliliter	ml	1 liter = 4 cups
cup	c	1 cup = 250 ml
tablespoon	T	1 T = 15 ml
teaspoon	tsp	1 tsp = 5 ml

Problem

Question A pitcher contains 2 qt (2 liters) of lemonade. How many cups (250 ml) can be filled with the lemonade?

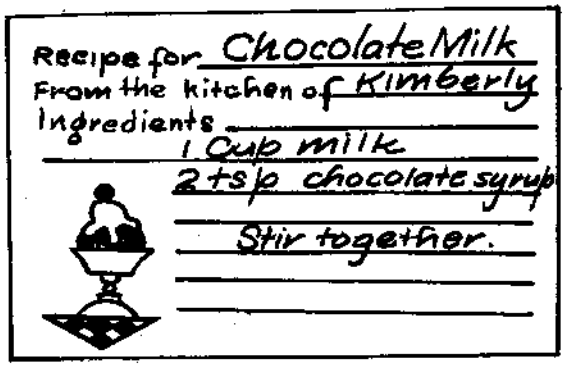
Facts:
2 qt = 2 liters
2 liters = 2000 ml
1 cup = 250 ml
? cups = 2000 ml

Think:
250 ml × ? = 2000 ml
250 ml × 8 = 2000 ml

Answer 8 cups

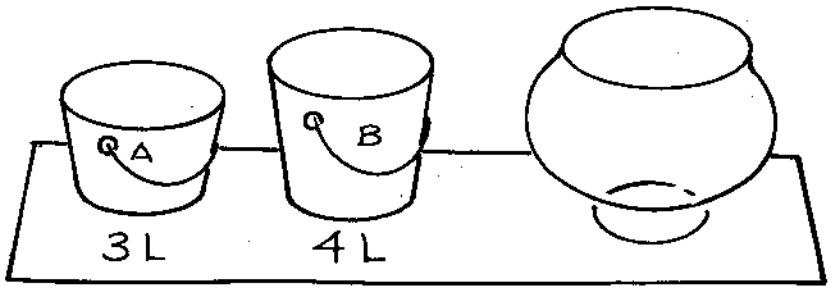
Exercises

1. Rewrite Kimberly's recipe for chocolate milk using milliliter measuring units.



2. Jennifer makes a 2 qt pitcher of orange juice by combining 500 ml of orange juice concentrate and water. How much water was added to fill the pitcher?

***3.** Lauren has been asked to pour 5 liters of water into the fishbowl by using the unmarked buckets. Describe a method that she can use to measure the liquid.



Activity: THE GLOB

Purpose To measure and use equivalent capacities.

GLOB SOLUTIONS
sodium borate solution 15 ml sodium borate + 1 liter water liquid adhesive solution 18 ml liquid adhesive + 18 ml water

EQUIVALENT GLOB SOLUTIONS
borax solution 1 T borax + 1 quart water glue solution 4 fl oz glue + 4 fl oz water

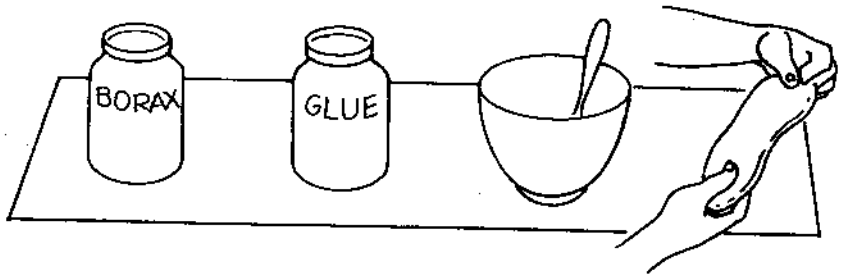
Materials

borax (sodium borate), found in supermarket with washing detergents
4 fluid oz bottle of liquid glue, white all-purpose
1 measuring spoon, tablespoon (15 ml)
1 measuring cup, 250 ml capacity
2 resealable plastic bags
distilled water
2-qt (2-liter) bowl
marking pen
2 clean, empty quart (liter) jars, with one tight-fitting lid

Procedure

- Make a borax solution by filling one of the qt (liter) jars with water. Label the jar Borax with the marking pen. Add 1 T (15 ml) of borax to the water. Put the lid on the jar and shake vigorously.
- Make a glue solution by emptying a 4 fluid oz bottle of glue into a second jar, labeled Glue. Fill the empty glue bottle with distilled water and pour the water into the jar. [Or measure out 4 ounces (18 ml) of glue and 4 ounces (18 ml) of water.] With a clean spoon, stir until the glue is thoroughly mixed.

- Pour 1 measuring cup (250 ml) of the borax solution into the empty bowl.
- Slowly pour the glue solution into the bowl containing the borax. Stir as you pour.
- Use the stirring spoon to dip the glob out of the bowl.
- Place the glob on top of a plastic bag for 2 minutes.
- Pick the glob up with your fingers and squeeze.
- Transfer the glob from one hand to the other, and squeeze until it and your hands are dry.



- Squeeze it! Pull it! Stretch it! Have fun with it!
- Place the glob inside the plastic bag and seal to store.
- Wash your hands when you are finished.

Results A white, pliable glob is formed that stretches and breaks easily when pulled apart sharply but flows if placed where gravity is the acting force.

You can make different colored globs by adding a drop of food coloring to the glue-water mixture.

Solutions

1. Facts

1 cup = 250 ml
1 tsp = 5 ml

Think!

1 cup milk = 250 ml milk
2 tsp = 2 × 5 ml = 10 ml chocolate syrup

2. Facts:

1 qt = 1 liter
1 liter = 1000 ml

Think!

2 qt = 2 liters
2 liters = 2000 ml

Think!

Volume of pitcher − Volume juice = Volume of water
2000 ml − 500 ml = 1500 ml

Answer 1500 ml of water

***3.** Facts:

1. Fill Bucket B with 4 liters of water.
2. Fill Bucket A with 3 liters of water from Bucket B.
3. Pour the 1 liter of water left in Bucket B into the fish bowl.
4. Fill Bucket B again and pour the 4 liters of water into the fishbowl to make a total of 5 liters.

18
Mass

Purpose To become more familiar with metric units and choose equivalent metric mass measurements.

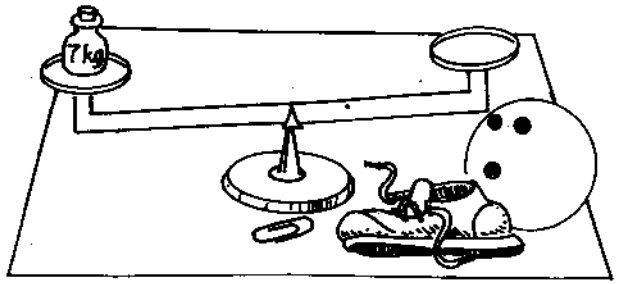
Facts Milligram (mg), centigram (cg), gram (g), and kilogram (kg) are metric units used to measure mass.

1000 milligrams (mg) = 1 gram (g)
100 centigrams (cg) = 1 gram (g)
1000 grams (g) = 1 kilogram (kg)

Problem

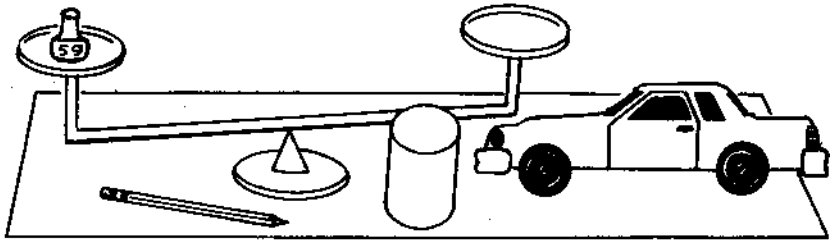
Question Choose the object that you think will balance the measured mass on the scale.

You must first make mental comparisons of the mass of each of the three objects. Which one has the greatest, medium, or least mass? Now evaluate the mass on the scale. Is 7 kg a large, medium, or small mass? Since kilogram is the largest of the three mass units used in this exercise, find an object that has a large amount of mass, such as the bowling ball.

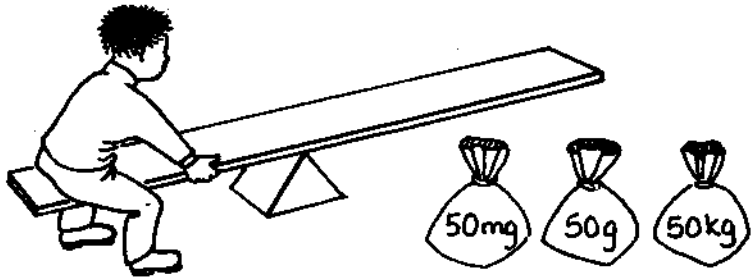


Exercises

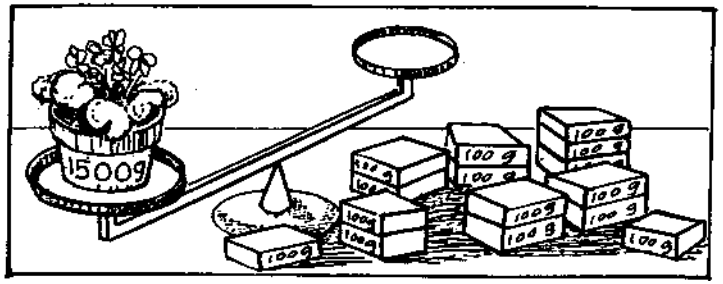
1. Which object has a 5 g mass?



2. Which of the bags will balance the child on the seesaw?



3. How many boxes of paper clips are needed to balance the 1500 g pot of flowers?



Activity: BALANCED

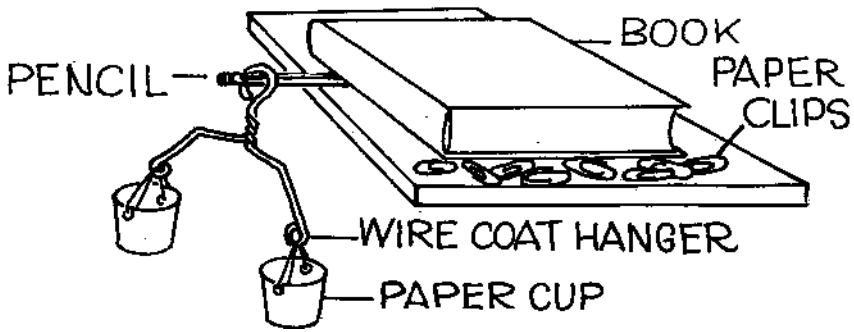
Purpose To make and use a balance to compare metric masses.

Materials

clothes hanger with paper rod	heavy book
2 paper cups	scissors
string	ruler
pencil	small paper clips
	coin

Procedure

- Cut two 12 in. (30 cm) lengths of string.
- Use the point of the pencil to make two holes in each of the paper cups. The holes should be near the top and on opposite sides of the cups.
- On each cup, tie one end of a 12 in. (30 cm) string in each hole to form a loop.
- Remove and discard the paper holder from the clothes hanger.
- Place the book on the edge of a table.
- Slip the end of the pencil under the book, leaving most of the pencil hanging over the edge of the table.
- Hang the clothes hanger on the pencil.
- Hang the string of each cup on one hook of the clothes hanger.



- Bend the wire arms up or down to make the two paper cups hang level.
- Place a coin in the left cup.
- Add paper clips to the right cup, one at a time, until the cups again are level.

Results The cups and string have an equal mass and balance on the arms of the clothes hanger. The number of paper clips needed to balance the coin will depend on the mass of the coin that you use.

Did You Know?

The brachiosaurus is believed to have been about 39 ft (12 m) tall, 75 ft (23 meters) long, and have a mass of 45,000 kg.

Solutions

1. Order

Largest mass = Car
Medium mass = Can of soup
Least mass = Pencil

Think! 5 grams is a small mass. A paper clip masses about 1 g, which object would be about the same mass as five paper clips?

Answer Pencil

2. Order

Largest mass = 50 kg
Medium mass = 50 g
Least mass = 50 mg

Think! 50 mg is the mass of about 5 fleas, and 50 g equals the mass of 50 paper clips. The child would be balanced by the bag massing 50 kg.

Answer 50-kg bag

3. *Think!

		One box of paper clips	= 100 g
		Mass of flower pot	= 1500 g
Mass of one box	x	Number of boxes	= Mass of flower pot
100 g	x	?	= 1500 g
100 g	x	15 boxes	= 1500 g

Answer 15 boxes

19
Weight

Purpose To become more familiar with the weight measurements of ton, pound, and ounce.

Facts

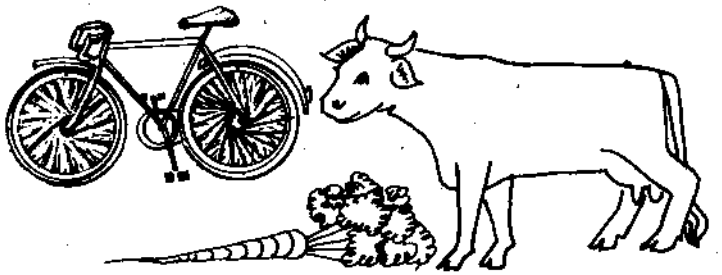
16 ounces (oz) = 1 pound (lb)
2000 pound (lb) = 1 ton (T)

Reference items:

Small car = 1 ton
Loaf of bread = 1 lb
Slice of cheese = 1 oz

Problem

Question Choose the item that weighs closest to 16 pounds.

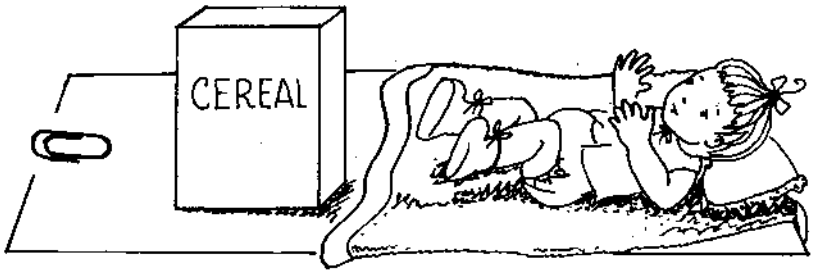


Question Mentally label the objects as largest, medium, and least weight. Compare each object with some known reference such as those in the Facts section of this exercise. Now evaluate the weight of 16 lb, which is the weight of 16 loaves of bread. The most likely choice would be the bicycle.

Exercises

1. Which object weighs 2 ounces?

2. Choose the object that weighs 8 pounds.

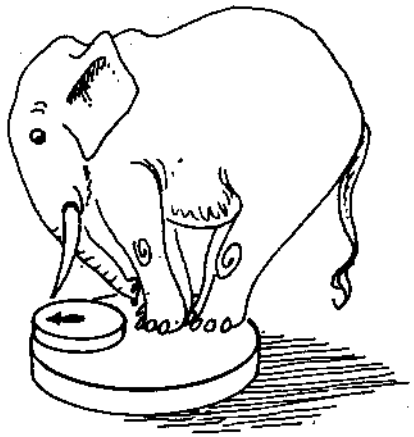


3. What is the weight of the elephant?

a. 2 tons

b. 2 ounces

c. 2 pounds



4. Three kittens were placed inside a basket to weigh them. If the empty basket weighs 4 pounds, how much does each kitten weigh, assuming they have equal weights?

Activity: FORCES

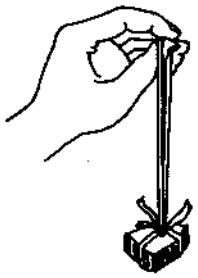
Purpose To demonstrate forces that affect weight.

Materials

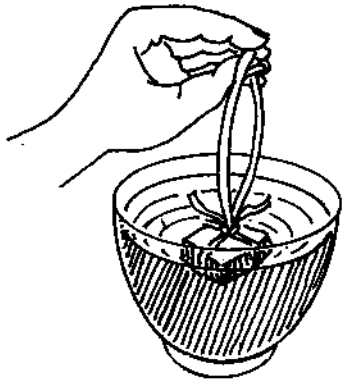
- large rock (a brick will work)
- string
- scissors
- rubber band
- bucket

Procedure

- Fill the bucket ¾ full with water.
- Tie the string around the rock.
- Cut the rubber band to form one long strip.
- Tie one end of the rubber band securely to the string around the rock.



- Hold the free end of the rubber band and slowly pull upward until the rock is suspended.
- Observe the length of the rubber band.
- Slowly lower the rock into the bucket of water until it is suspended about in the middle of the water.



- Again observe the length of the rubber band.

Results The rubber band is much longer when the rock is suspended in the air than when it is in water. A force called gravity pulls the rock toward the earth; the amount of this pull is called the weight of the rock. The upward push of air changes the weight some, but the upward push of the water makes a more noticeable change in the weight of the rock.

Did You Know?

Weight is a result of the pull of gravity. Other celestial bodies have gravity, but in varying amounts. The chart indicates a person's weight on different bodies in our solar system.

Where	Weight
Earth	100 lb
Moon	17 lb
Sun	27,900 lb
Mars	38 lb

Divide your weight by 6 to determine your weight on the moon.

Solutions

1. Order
Largest weight = Hippopotamus
Medium weight = Jar of jelly
Least weight = Slice of bread

Think! 2 oz is a small weight. A slice of cheese weighs 1 oz. Which object weighs the same as two slices of cheese?

Answer The slice of bread.

2. Order
Largest weight = Baby
Medium weight = Cereal
Least weight = Paperclip

Think! The ton unit will not be used since none of the three objects weights as much as a car. This leaves the pound and ounce units as choices for each object's weight. Since a loaf of bread weighs 1 lb, which of the three items weighs as much as 8 loaves of bread?

Answer The baby.

3. Order
Largest weight = 2 tons
Medium weight = 2 lb
Least weight = 2 oz

Think! You know that 2 oz is the weight of two slices of cheese and that two loaves of bread weigh 2 lb.

Answer The elephant must weigh 2 tons.

4.

Weight of the basket + Weight of the three kittens	= 10 lb
− Weight of the basket	= − 4 lb
Weight of the three kittens	= 6 lb

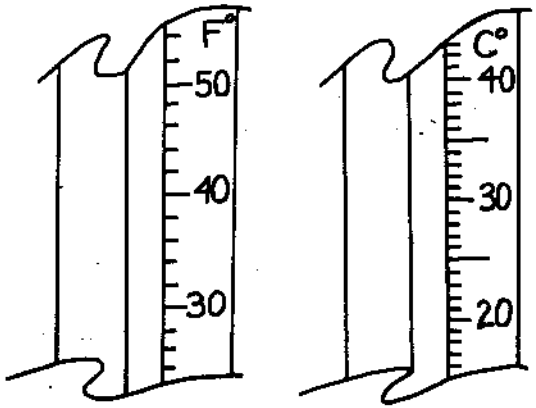
Think!
Weight of the three kittens = 6 lb
Weight of 3 kittens × ? lbs = 6 lb
3 × 2 lb = 6 lb

Answer Each kitten weighs 2 lb.

20
Temperature

Purpose To read Fahrenheit and Celsius thermometers.

Facts The two types of thermometers used in this exercise are Celsius and Fahrenheit. Notice that the Fahrenheit scale has five divisions between each printed number. Each division equals 2 degrees. There are 10 divisions between each printed number on the Celsius scale. Each division equals 1 degree. The 5th mark on the Celsius scale is longer to make it simpler for you to find the mid-point.



The symbol for the word degree is a small raised circle. °F is read as degrees Fahrenheit and °C is read as degrees Celsius.

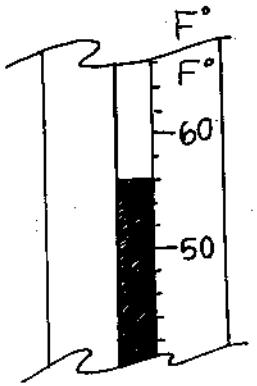
Examples: 30°C is read as 30 degrees Celsius
40°F is read as 40 degrees Fahrenheit

Problem

Question Take temperature readings from the two thermometers.

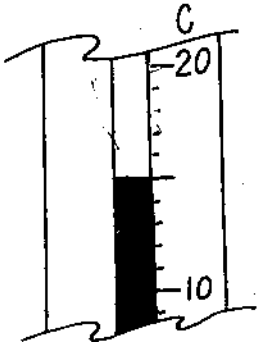
Think! **Fahrenheit Scale**
The height of the liquid in the thermometer is at the third mark above 50°F. Each mark equals 2°F.

Thus, the reading on the thermometer is 56°F.



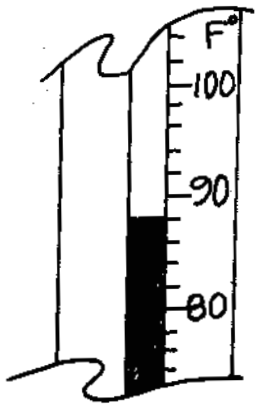
Celsius Scale
The height of the liquid in the thermometer is at the fifth mark above 10°C. Each mark equals 1°C.

Thus, the reading on the thermometer is 15°C.

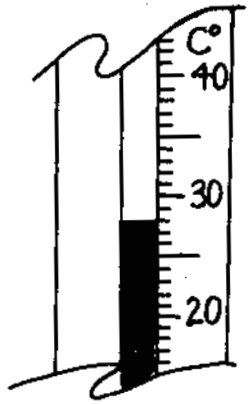


Exercises

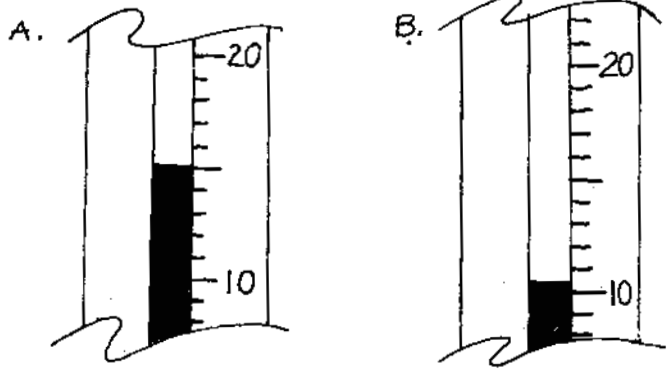
1. Read the thermometer.



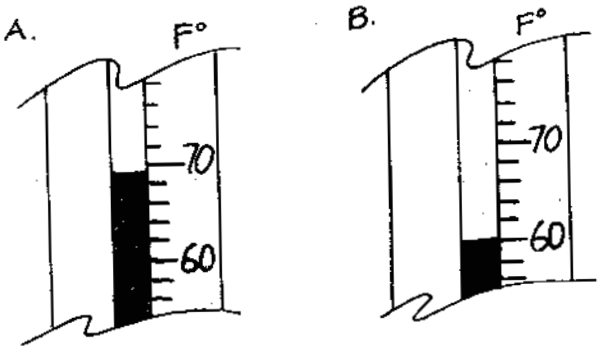
2. Read the thermometer.



3. Which thermometer reads 10.5°C?

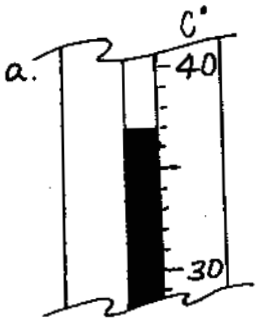


4. Which thermometer reads 69°F?

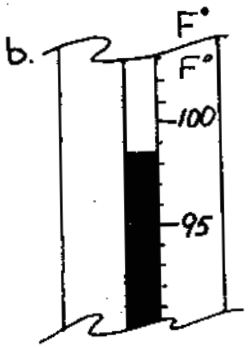


5. Use the labels on the diagram to find the temperature of each of the following.

a. Body temperature of humans in degrees Celsius



b. Body temperature of humans in degrees Fahrenheit



Activity: STRAW THERMOMETER

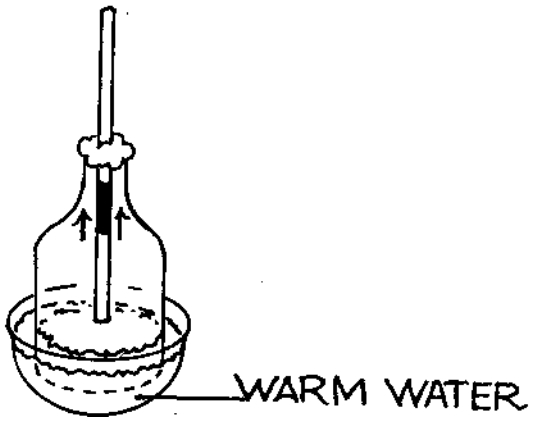
Purpose To demonstrate how a thermometer works.

Materials

- blue food coloring
- straw, small, with little or no color
- modeling clay, walnut-sized piece
- 2 soup bowls large enough to hold the bottle
- 2 ice cubes
- glass soda bottle
- measuring cup (250 ml)

Procedure

- Half fill the measuring cup (125 ml) with water.
- Add drops of blue food coloring to the water and stir. Continue adding the coloring until the water is deep blue.
- Place the end of the straw into the colored water.
- While the straw is in the water, place your index finger over the open end of the straw.
- Hold the straw closed with your finger while lifting the straw out of the colored water and inserting the free end in the empty soda bottle.



Using your other hand, seal around the mouth of the bottle with clay before removing your finger from the end of the straw.

The colored water plug will move up the straw when you release the end. If it comes out the top, try again. If it falls out the bottom, use a straw with a smaller diameter. You want the water to stay in the straw.

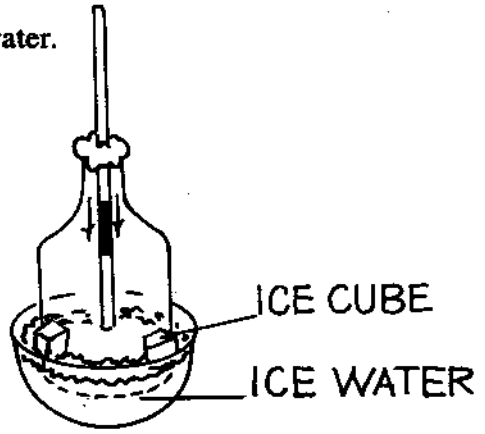
Fill one of the soup bowls half full with warm water from the faucet.

The second bowl is to be filled half full with cold water from the faucet. Add the two ice cubes to this water.

Place the empty soda bottle with the straw in the warm water.

Remove the bottle from the water when the colored water in the straw starts to move.

Place the bottle in the ice water.



Again remove the bottle when the colored water in the straw starts to move.

Results The colored water moves up when the bottle is placed in warm water and down when the bottle is in cold water. The air inside the bottle expands when heated. This expanding gas pushes the cold water up the straw. Cooling the air inside the bottle causes it to contract, and the air above the straw pushes the colored water down. Liquids inside thermometers are in a closed tube. When the liquid is heated, it expands and moves up. Cooling the liquid causes contraction, and the liquid moves down the tube.

Did You Know?

The average person has a body temperature of 98.6°F (37°C). A human cannot live with a body temperature above 109°F (42.8°C) or lower than 95°F (35°C). Marathon runners in hot weather have been known to attain body temperatures of 105.8°F (41.°C).

Solutions

1. ***Think!*** The height of the liquid in the thermometer is at the fourth mark above 80°F. Each mark equals 2°F.

Thus, the reading on the thermometer is 88°F.

2. ***Think!*** The height of the liquid in the thermometer is at the eighth mark above 20°C. Each mark equals 1°C.

Thus, the reading on the thermometer is 28°C.

3. Thermometer B reads 10.5°C.

Think! The height of the liquid in thermometer A is at the fifth mark above 10°C. Since each mark equals 1°C the temperature reading is 15°C.

The height of the liquid in thermometer B is halfway between 10°C and 11°C and can be read as 10½ or 10.5°C.

4. Thermometer A reads 69°F.

Think! The height of the liquid in thermometer A is halfway between 68°F and 70°F and is read as 69°F.

The height of the liquid in thermometer B is halfway to the first mark. Since each mark equals 2 degrees, the temperature would be a little less than 61°F. or about 60.9°F.

5. a. Body temperature of humans = 37°C.

b. Body temperature of humans = 98.6°F.

III
Graphing

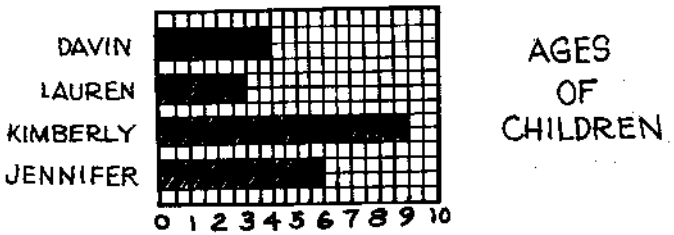
21
Bar Graph

Purpose To interpret information on a bar graph.

Facts Bar graphs make comparing information easier. Each box on the graph has the same value, and the starting value is always 0, not 1.

Problem

Question Use the vertical bar graph to answer Questions through 5.



1. Who is older, Davin or Jennifer?

2. Which children are older than Davin?

3. Name the children who are younger than Jennifer.

4. Who is the oldest?

5. List the ages of the children.
   The numbering on the horizontal scale indicates that a length of 2 boxes equals 1 year. A length of one box would then equal ½ of a year.

1. ***Think!*** Who has the longer bar, Davin or Jennifer?

   Answer Jennifer

2. ***Think*** Who has bars longer than Davin's bar?

   Answer Kimberly and Jennifer

3. ***Think!*** Who has bars shorter than Jennifer's bar?

   Answer Davin and Lauren

4. ***Think!*** Who has the longest bar?

   Answer Kimberly

5. ***Think!*** Start at the right end of each child's bar, and follow th line it touches all the way to the bottom to find th number on the scale.

   Answer

Davin	4 years
Lauren	3 years
Kimberly	9 years
Jennifer	6 years

Exercises

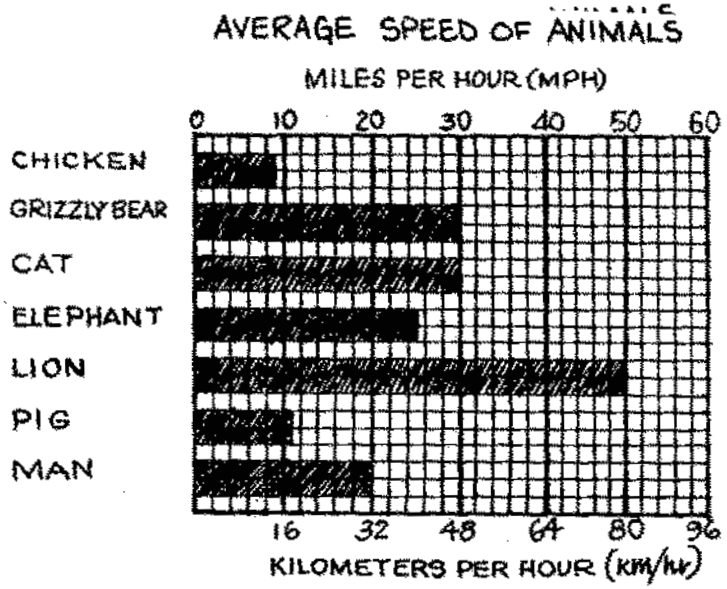
1. Use the following bar graph to answer Questions a through f. D termine all speeds in miles per hour and in kilometers per hour

   a. How much speed does each square measure?

   b. Which is the fastest animal?

   c. What is the difference in the speed of the lion and the cat?

   d. How many animals are slower than man?



   e. Which animals have the same speed?

   f. How many animals are faster than the pig?

2. Use the following bar graph to answer Questions a through f. f.

   a. How many years does each square measure?

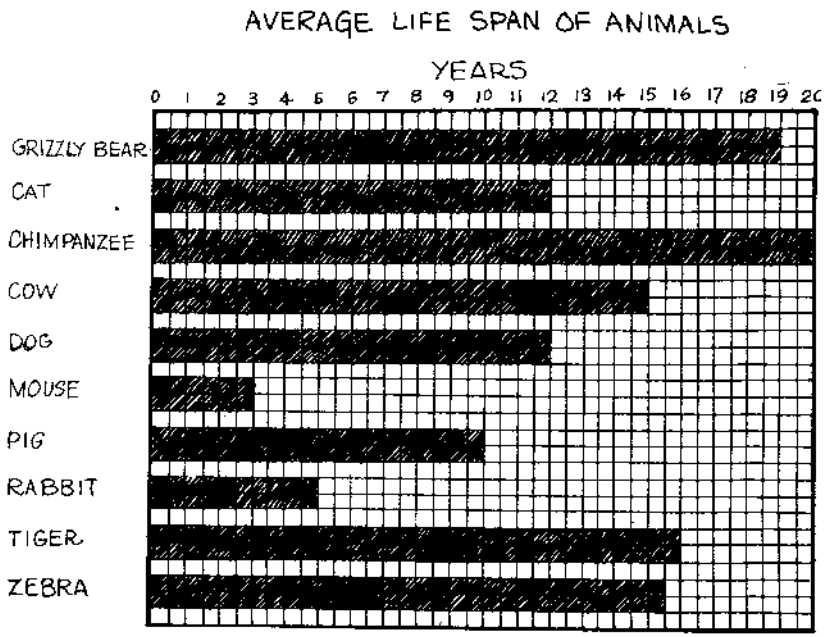
   b. Which animal has the shortest life span?

   c. How many animals live longer than a zebra?

   d. How many times longer does the bear live than the pig? ?

   e. Which animal lives twice as long as the rabbit?

   f. How many animals have the same life span as a dog?



Activity: BEAN GROWTH

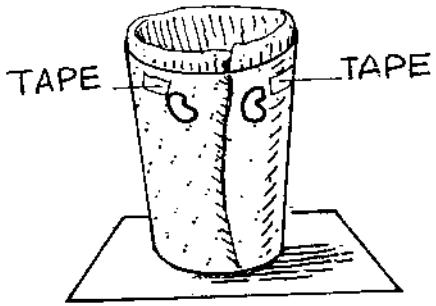
Purpose To graph the growth of a bean.

Materials

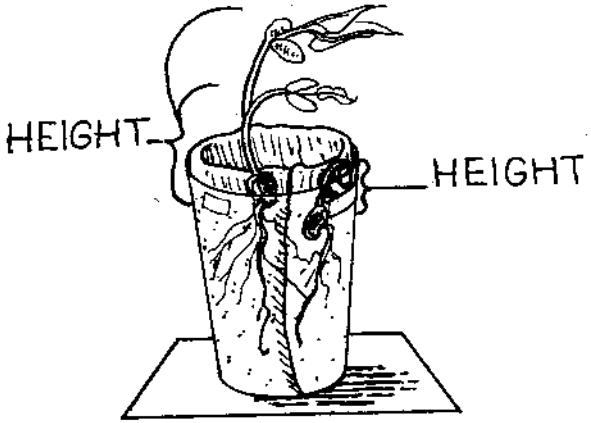
4 pinto beans	ruler
paper towels	note pad
masking tape	pencil
1 drinking glass	

Procedure

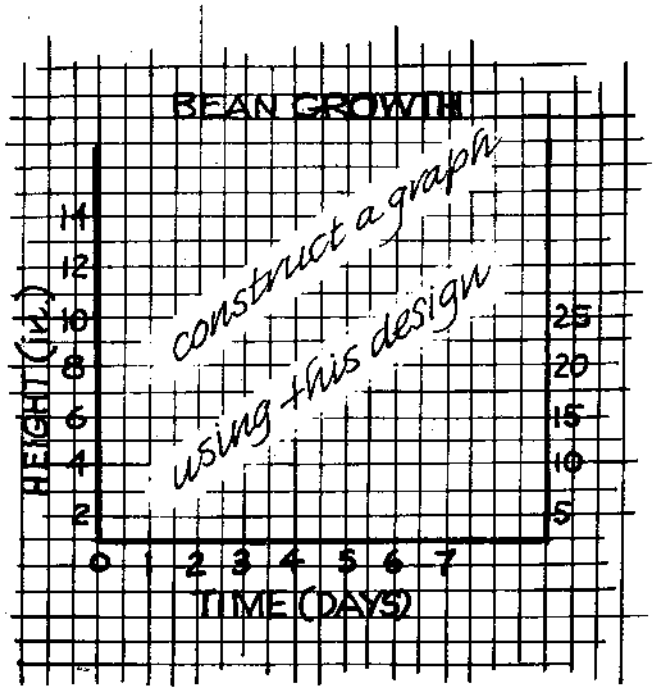
- Fold a paper towel, and line the inside of the glass with it.
- Wad sheets of paper towels and stuff them into the glass to hold the paper lining tightly against the glass.
- Place the beans between the paper lining and the glass. The beans should be evenly spaced and about 1 in. (2.5 cm) from the top of the glass.



- Moisten the paper towels in the glass with water. You do not want the paper to be dripping wet, only moist.
- Keep the paper moist and observe each day until the beans start to grow.
- When the first sign of a leaf appears, place a piece of tape on the outside of the glass to mark the position of the top of the leaf. The tape marks the beginning of the growth measurements.
- Measure the growth from the tip of the leaf to the top of the tape after 24 hours (1 day). Record as the growth for day 1.



- Continue to measure and record the growth for 7 days.
- Use the measurements to construct a bar graph.



Results It takes about 7 days before the bean starts to grow. Within another 7 days, the plant growth is very quick. Many centimeters of growth can occur overnight. The bar graph provides an easy way to picture determine how fast the bean grows each day.

Did You Know?

The tallest recorded height of a sunflower plant is 24 ft 2½ in. (7.45 m).

Solutions

1. a. The numbering on the horizontal scales indicate that each square equals 2 mph (3.2 km/hr).

b. ***Think!*** Which animal has the longer bar?

Answer Lion

c. ***Think!*** Start at the right end of the bars for the lion and the cat. Follow the lines up and down to find the numbers on the scales.

Speed of the lion	=	50 mph (80 km)/hr
− Speed of the cat	=	− 30 mph (48 km)/hr
Difference	=	20 mph (32 km)/hr

Answer The lion can run 20 mph (32 km/hr) faster than a cat.

d. ***Think!*** How many bars are shorter than the one for man?

Answer Two; the chicken and the pig

e. ***Think!*** How many bars are exactly the same length?

Answer Two; the bear and the cat

f. ***Think!*** How many bars are longer than that of the pig?

Answer Five; the bear, cat, elephant, lion, and man

2. a. Each square measures one half year.

b. ***Think!*** Which bar is closest to the left?

Answer Mouse

c. ***Think!*** How many bars are farther to the right than is the bar representing zebras?

Answer Three; the bear, chimpanzee, and tiger

d. Determine the life span of the bear and the pig by following each bar to the right and then down to the scale. Take the difference between the numbers.

$$\begin{array}{rcr} \text{Life span of the bear} & = & 19 \text{ years} \\ -\ \text{Life span of the pig} & = & -\ 10 \text{ years} \\ \hline \text{Difference} & = & 9 \text{ years} \end{array}$$

Answer The bear lives 9 years longer than the pig.

e. ***Think!*** How many bars extend to the right exactly the same distance as that of the dog?

Answer One; the bar of the cat

f. Determine the life span of the rabbit by following the rabbit's bar to the right and then down to the scale.

$$\text{Life span of the rabbit} = 5 \text{ years}$$

Multiply the rabbit's life span by 2.

$$2 \times 5 \text{ years} = 10 \text{ years}$$

Find 10 years on the scale and move upward until you reach a bar that ends on the 10-year line.

Answer Pig

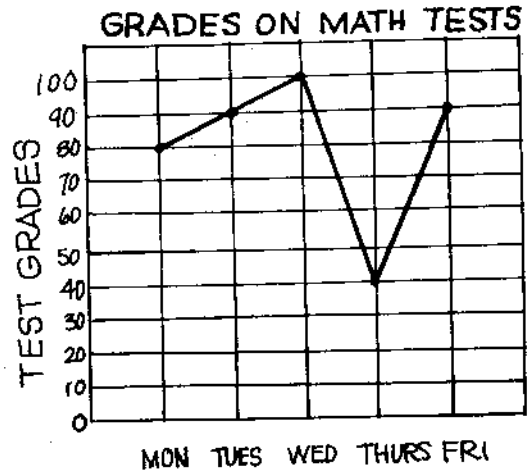
22
Line Graph

Purpose To interpret information on a line graph.

Facts Information is recorded as points on the graph. A line is formed by joining the points in order starting on the left and moving to the right. It is not always necessary to start from zero on a line graph.

Problem

Question For one week, Jennifer took a daily math test. The scores from these tests are recorded on the line graph. Use the graph to answer Questions 1 through 3.



1. On which day did Jennifer know the most answers?

2. On which day did she know the fewest answers?

3. On which night did Jennifer watch television instead of studying for her math test?

1. ***Think!*** Which day has the highest point on the line?

Answer Wednesday

2. ***Think!*** Which day has the lowest point on the line?

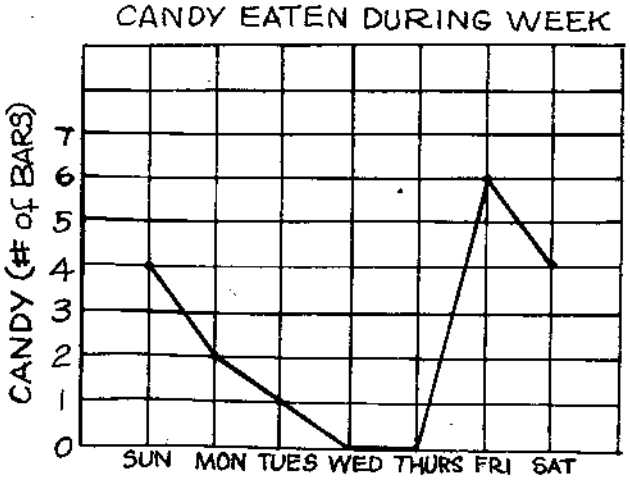
Answer Thursday

3. ***Think!*** Thursday was the day she scored the lowest grade. We can thus assume that she watched television on Wednesday night instead of preparing for her test.

Answer: Wednesday night

Exercises

1. Robert buys candy with his allowance money. The number of candy bars he ate during 1 week is recorded on the line graph. Use the graph to answer Questions a through d.



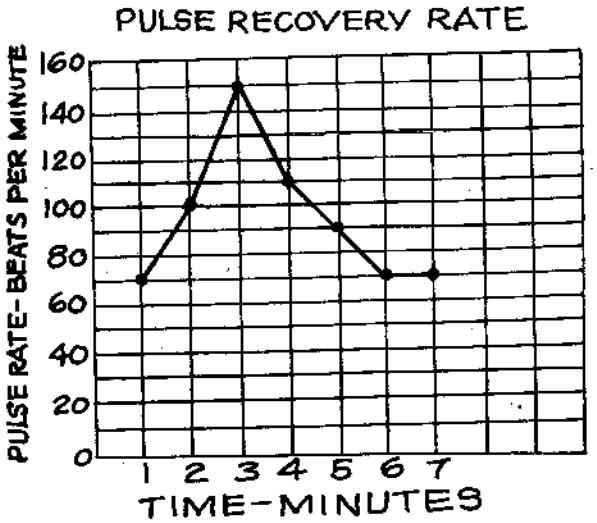
a. On which day were the most candy bars eaten?

b. On which days were more than three bars of candy eaten?

c. How many days did Robert not eat candy?

d. How many bars of candy did Robert eat during the week?

2. Russell timed his pulse rate before and after exercising to determine his recovery pulse rate, the time it took for his pulse rate to return to normal. His pulse rates are recorded on the line graph. Use the graph to answer Questions a through d.



a. What is Russell's normal pulse rate?

b. How long did he exercise?

c. What was his highest pulse rate?

d. How long did it take his pulse to return to normal?

Activity: HOW FAST?

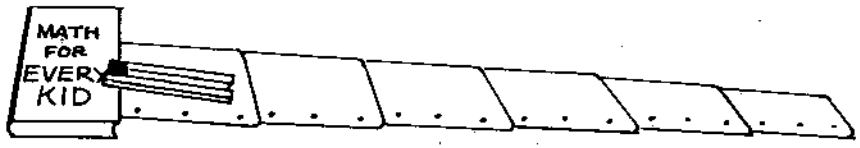
Purpose To use a distance versus time line graph to compare the speed of a moving object at different times.

Materials

- ruler
- book
- timer, second hand on a watch
- notebook paper
- marble
- pencil
- helper

Procedure

- Place 6 sheets of paper on the floor to form a long path.
- Position the book on the edge of the paper trail.



- Rest one end of the ruler on the edge of the book and the other end on the paper.
- Hold the marble at the top of the ruler.
- Release the marble, allowing it to roll down the center groove in the ruler.
- Your helper should be ready with a pencil to mark the position of the marble as it rolls across the paper.
- Start timing as soon as the marble touches the paper.
- Count aloud the passing of each second until 4 seconds have passed.
- As each second is announced, your helper should mark on the paper the position of the marble.
- Measure and record the inch and centimeter distances from the end of the ruler to each mark.

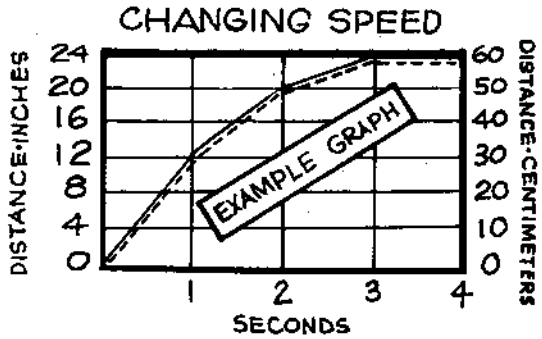
DATA TABLE #1

TIME SECONDS	DISTANCE INCHES	SPEED IN SECONDS

DATA TABLE #2

TIME SECONDS	DISTANCE INCHES	SPEED IN SECONDS

- Plot the data on a line graph. Use a solid line to connect inch measurements and a dashed line for the centimeter distances.



- Observe the steepness of the lines between points.

Results The marble moves a greater distance during the first second because it is traveling at a faster speed. The speed decreases until finally the marble stops. This decrease in speed is indicated by the decrease in distance traveled. The height of the line between points indicates the speed. The steeper the line, the faster the speed. A horizontal line indicates no change in distance, thus a speed of zero.

Did You Know?

The marble slows and stops due to friction. Friction is a resistance to movement. Without friction, the marble would have continued to move until it hit another object. This is what happens to moving objects in space.

Solutions

1. a. ***Think!*** Which day has the highest point on the line?

Answer Friday

b. ***Think!*** Which days have points higher than the horizontal line representing 3 candy bars?

Answer Sunday, Friday, and Saturday

c. ***Think!*** Which days have points on the horizontal line representing 0 candy bars?

Answer Two; Wednesday and Thursday

d. ***Think!*** How many candy bars did he eat each day? Add them up to find how many he ate that week.

Answer $4+2+1+6+4 = 17$

2. a. ***Think!*** What was the beginning and ending pulse rate?

Answer 70 beats per minute

b. ***Think!*** When did the rate start to increase and when did it stop? It began increasing at 1 minute and stopped increasing at 3 minutes. How many minutes passed between these points?

Answer Two minutes

c. ***Think!*** What is the highest point on the line? Follow this point to the left scale and read the number.

Answer 150 beats per minute

d. ***Think!*** At what time did the rate start to change and when did it return to normal—70 beats per minute? It started increasing at 1 minute and returned at 6 minutes. How long between these time periods?

Answer 5 minutes

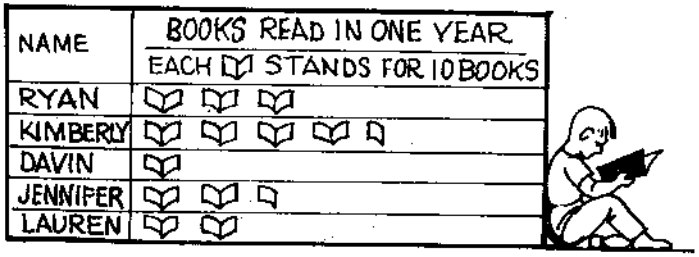
23
Pictograph

Purpose To interpret and construct pictographs.

Facts Pictographs contain symbols that represent a specific number of objects. In the problem below, a book is used as a symbol to represent 10 books that had been read. Pictographs are fun and easy to read. Symbols show data involving large or small numbers.

Problem

Question The number of books read by five different people was recorded on a pictograph. Use the pictograph to answer Questions 1 through 3.



a. Who read the fewest books?

b. How many books did Jennifer read?

c. How many more books must Davin read to equal the books read by Jennifer?

a. ***Think!*** Which name has the fewest book symbols?

Answer Davin

b. ***Think!*** How many symbols follow Jennifer's name? There are 2½ symbols. The half symbol would equal half of 10 or 5.
2½ symbols = 20 + 5 = 25 books

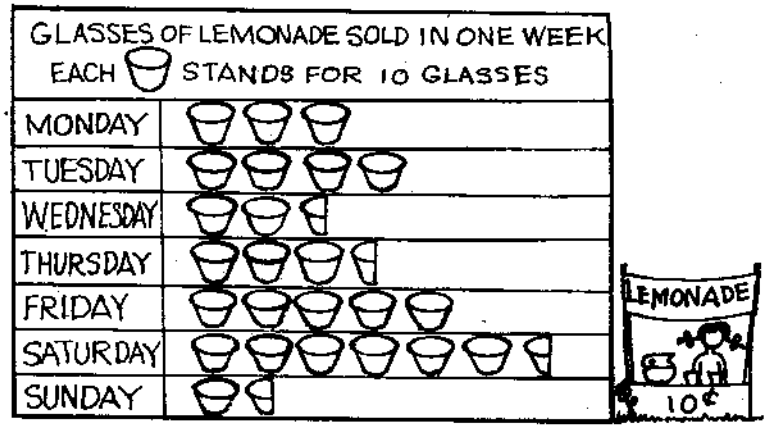
Answer 25 books

c. ***Think!*** How many symbols follow Davin's name? The 1 symbol indicates that he read 10 books.
10 books + ? = 25 books read by Jennifer

Answer 15 books

Exercises

1. A group of children opened a lemonade stand. They sold each glass of lemonade for 10 cents each. Use the pictograph to answer Questions a through e about the lemonade sales.



a. On which day were the most glasses of lemonade sold?

b. Lauren drank the unsold glasses of lemonade on Saturday. If 70 glasses of lemonade were prepared for Saturday, how many glasses did she drink?

c. On which two days were the most sales made?

d. On which day was the least amount of money made from sales?

***e.** How much money was collected from the sales of the lemonade?

2. A balloon was given to each person attending a school carnival. Use the pictograph to answer Questions a through d.

DAY	CARNIVAL ATTENDANCE (B STANDS FOR 50 PEOPLE)
FRIDAY	B B B B B
SATURDAY	B B B B B B B
SUNDAY	B B B B

a. On which day were 250 balloons given away?

b. How many people attended the carnival during the 3 days?

c. 400 balloons were prepared for Saturday. Were there enough balloons?

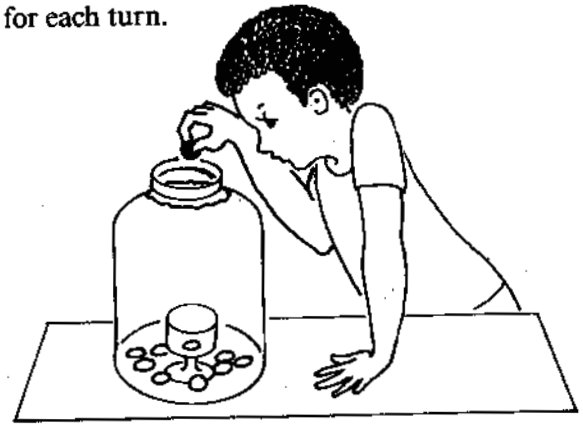
Activity: COIN DROP

Purpose To collect data and record it as a pictograph.

Materials
1 gallon (4 liter) jar
egg holder or cup
10 coins
paper
pencil

Procedure

- Center the egg holder in the bottom of the jar.
- Fill the jar with water.
- Hold one coin at a time above the water's surface.
- Drop each coin so that it falls through the water and into the egg holder.
- Use 10 coins for each turn.



- Draw a pictograph on the paper. Record the number of coins that fall into the egg holder during 10 turns, using a circle to represent each coin in the holder.

TURN	COINS DROPPED IN HOLDER (○ STANDS FOR ONE COIN)
1	
2	EXAMPLE
3	
4	

Results The coins fall straight down through the air and flip to the side when they hit the water's surface. A position above the water can be found to allow more coins to fall into the egg holder.

Turn	Coins Dropped in Holder ◯ Stands for one coin.
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Did You Know?

Light changes direction when it enters water, as did the coins. This change in the direction of light is called refraction and causes objects in water to appear to be where they are not.

Solutions

1. a. ***Think!*** Which day has the largest number of glass symbols?

Answer Saturday

b. ***Think!*** Find the difference between the number of glasses prepared and the number of glasses sold. How many were sold? How many prepared?
prepared (70) – sold (65) = 5 glasses

Answer Lauren drank 5 glasses of lemonade.

c. ***Think!*** Which two days have the most symbols?

Answer Friday and Saturday

d. ***Think!*** On which day were the least sales made? This will be the day the least amount of money was made.

Answer Sunday

e.** ***Think! How many total glasses were sold? Count all of the symbols and multiply by the cost of one glass, 10 cents. 26 symbols × 10 = 260

260 glasses × \$0.10 = \$26.00

Answer \$26.00

2. a. ***Think!*** Number of people - Number of balloons
? symbols × 50 = 250
? = 5 symbols

Thus, the day having 5 balloon symbols is the day that 250 balloons were given away.

Answer Friday

b. ***Think!*** ? balloon symbols × 50 = Total attendance
16 × 50 = 800 people

Answer 800 people attended the carnival.

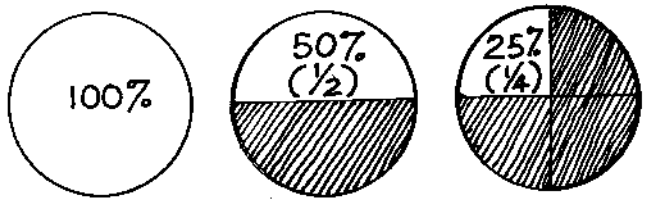
c. ***Think!*** How many people received balloons on Saturday?
7 symbols × 50 = 350 people
400 balloons − 350 balloons = 50 balloons

Answer Yes, there were 50 extra balloons.

24
Circle Graph

Purpose To interpret information on a circle graph.

Facts Information on circle graphs is usually shown as a percentage. The larger the area of the graph used, the greater the percentage represented. The whole circle represents 100 percent or the total amount. Dividing the circle in half makes each section equal to 50 percent, and four equal divisions produce 25 percent portions.



Percentage is a special ratio that compares a number to 100. The percent symbol, %, means hundredths. 60% is read as "sixty percent" and means $^{60}/_{100}$, sixty hundredths. Percent numbers can be expressed as a decimal number by dividing the numerator by 100. Thus 60% is the same as $^{60}/_{100}$ or .60.

Problems

Question **1.** Twenty children were asked about their favorite snacks. Find the number of children that ate each type of food.

a. 35% like potato chips

b. 60% like candy

c. 5% like raisins

a. ***Think!*** 35% = $^{35}/_{100}$ = .35
35% of 20 children = .35 × 20 = 7 children

Answer 7 children like potato chips

b. ***Think!*** 60% = $^{60}/_{100}$ = .60
60% of 20 children = .60 × 20 = 12 children

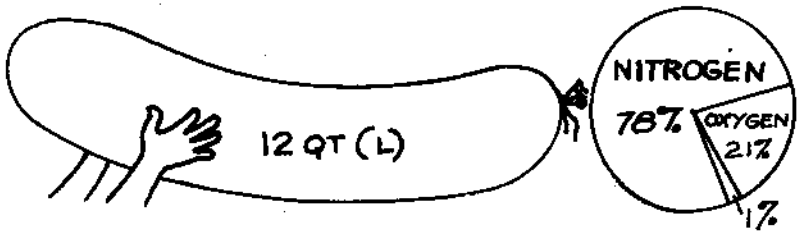
Answer 12 children like candy

c. ***Think!*** 5% = $^{5}/_{100}$ = .05
5% of 20 children = .05 × 20 = 1 child

Answer 1 child likes raisins

Note: The sum of the number of children who like the different snacks is equal to 20, the total number of children.

2. Use the circle graph to answer Questions 1 through 3 about the 12 liters of air inside the balloon.



a. What is the sum of all the percentages in the graph?

b. What percentage of the air in the balloon is nitrogen?

c. How many liters of oxygen gas are in the balloon?

a. ***Think!*** 78% + 21% + 1% = 100%

Answer 100% is the sum of all the percentages in any circle graph.

b. ***Think!*** Find the part of the circle labeled nitrogen.

Answer 78%

c. ***Think!*** 21% of the volume of air in the balloon is oxygen.
thus
21% of 12 qt (12 liters) = .21 × 12 qt
= 2.52 qt (2.52 liters)

Answer 2.52 qt (2.52 liters) of oxygen in the balloon

Exercises

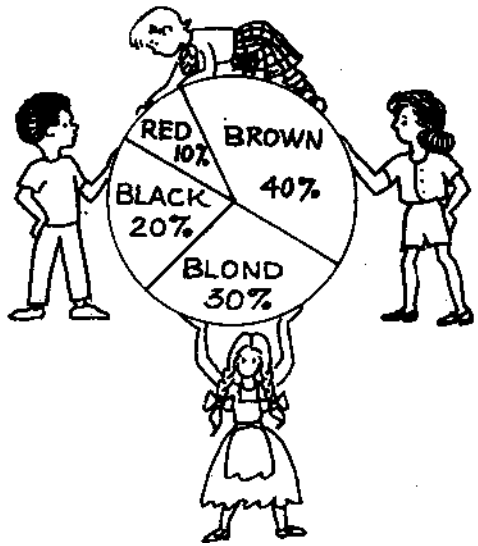
1. The circle graph shows the percentage of hair color in a class of 30 students. Use the graph to determine how many students have each hair color.

a. brown

b. blond

c. black

d. red



2. Davin divided his daily homework time of 60 minutes. Use the graph to determine how many minutes he now spends on the separate subjects.

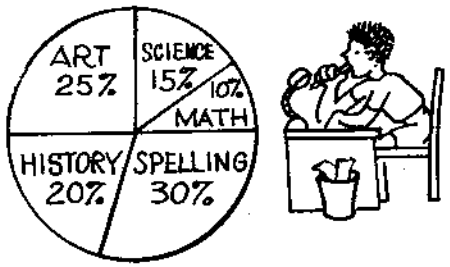
a. math

b. art

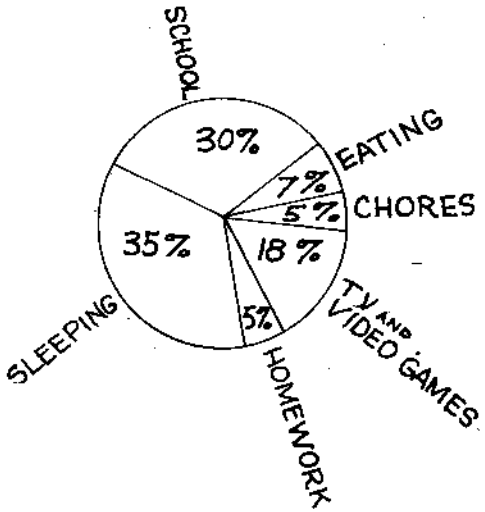
c. history

d. science

e. spelling



3. The circle graph is a record of how Ryan spends his time in a 24-hour day.



a. How much sleep does Ryan get each week?

b. How much time does he spend studying if he only studies on week nights?

Activity: COLOR WHEEL

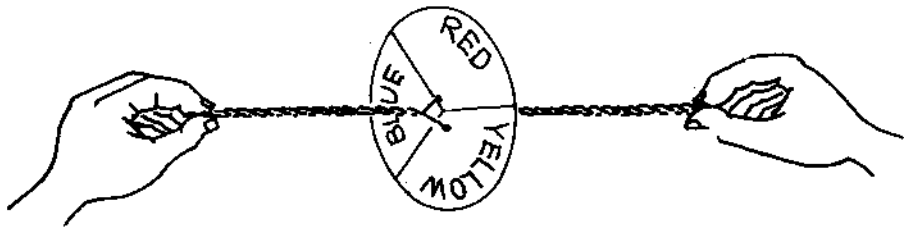
Purpose To use a circle graph of colors to demonstrate blending of colors.

Materials

- cardboard
- tempera paints, red, blue, and yellow
- paint brush
- straight pin
- ruler
- pencil
- glue

Procedure

- Cut an 8 in. (20 cm) diameter circle from the stiff paper.
- Use the pencil to draw lines to divide the paper circle into three equal parts. Each part represents 33⅓%.
- Color each section of the circle a different color, red, blue, and yellow.
- Allow the paint to dry.
- Use the pin to make two holes in the center of the circle about ½ in. (1 cm) apart.
- Cut a 24 in. (60 cm) length of string.
- Run the string through one of the holes and back through the other one.
- Tie the ends of the string together.
- Move the paper circle to the center of the string.
- Turn the circle around until the string is tightly twisted.
- Pull outward on both ends of the string until it starts to unwind, then release the tension on the string so that it winds in the opposite direction.



- Continue to pull and release the string causing the circle to quickly spin back and forth.

Results The paper circle quickly spins back and forth. The colors blend together and a grayish color is seen.

Did You Know?

Your brain retains each color 1/16 of a second after the section has passed, causing a blending of the colors seen. If the colors used to paint the paper were pure blue, red, and yellow, the color of the spinning paper would be white instead of grey.

Solutions

1. a. ***Think!*** 40% of 30 children = number with brown hair
$40\% \times 30 = .40 \times 30 = 12$ children

Answer 12 children with brown hair

b. ***Think!*** 30% of 30 children = number with blond hair
$30\% \times 30 = .30 \times 30 = 9$ children

Answer 9 children with blond hair

c. ***Think!*** 20% of 30 children = number with black hair
$20\% \times 30 = .20 \times 30 = 6$ children

Answer 6 children with black hair

d. ***Think!*** 10% of 30 children = number with red hair
$10\% \times 30 = .10 \times 30 = 3$ children

Answer 3 children with red hair

2. a. ***Think!*** $10\% \times 60$ minutes = study time for math
$10\% \times 60 = .10 \times 60 = 6$ minutes

Answer 6 minutes study time for math

b. ***Think!*** $25\% \times 60$ minutes = study time for art
$25\% \times 60 = .25 \times 60 = 15$ minutes

Answer 15 minutes study time for art

c. ***Think!*** $20\% \times 60$ minutes = study time for history
$20\% \times 60 = .20 \times 60 = 12$ minutes

Answer 12 minutes study time for history

d. ***Think!*** $15\% \times 60$ minutes = study time for science
$15\% \times 60 = .15 \times 60 = 9$ minutes

Answer 9 minutes study time for science

e. ***Think!*** $30\% \times 60$ minutes = study time for spelling
$30\% \times 60 = .30 \times 60 = 18$ minutes

Answer 18 minutes study time for spelling

3. a. ***Think!*** There are 7 days in a week. Multiply the time spent in sleep in 1 day by 7.
$35\% \times 24$ hours $= .35 \times 24$ hours $= 8.4$ hours
8.4 hours $\times 7 = 58.8$ hours per week

Answer 58.8 hours of sleep each week

b. ***Think!*** Weekdays are Monday through Friday, thus he studies 5 days each week.
$5\% \times 24$ hours $= .05 \times 24 = 1.2$ hours
1.2 hours $\times 5 = 6$ hours per week

Answer 6 hours of studying each week

25
Graphs

Purpose To use data charts to construct graphs.

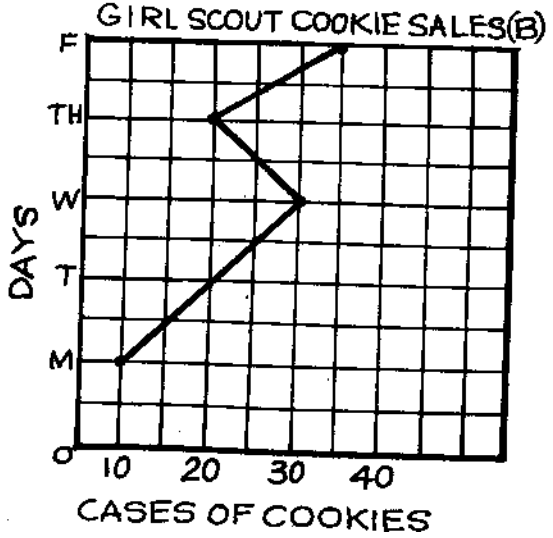
Facts Each division on the graph must have the same value. The vertical and horizontal scales are labeled to indicate what they measure. A title explains what is being graphed. The type of graph depends on the data collected. When one factor varies while a second factor changes, such as the sale of the cookies each day in the example, a line graph would best represent the data. Bar graphs are used to show comparisons between data. Circle graphs best represent fractions, and percentages and pictographs can be used as score cards or when large quantities are involved. Graph paper makes constructing a graph easier since the squares are uniform.

Problem

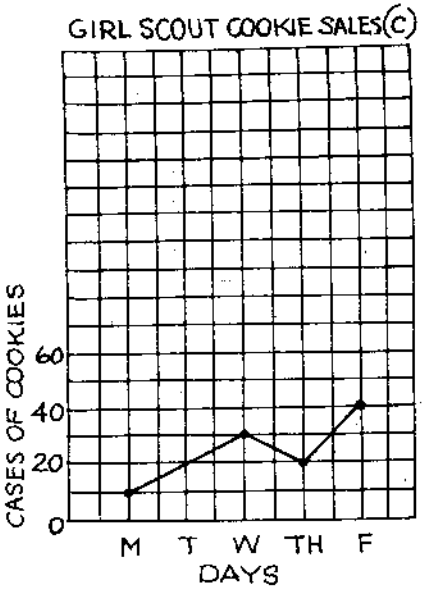
Question Use the following Girl Scout Cookie Sales data chart to construct a line graph.

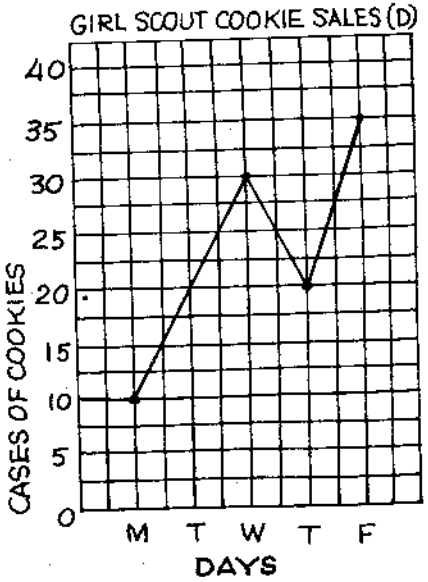
Girl Scout Cookie Sales	
Day	Number of Cases Sold
Monday	10
Tuesday	20
Wednesday	30
Thursday	20
Friday	35

Graphs A and B use the same value for the scales, but the position of the information is different. A line graph is used when one factor varies while the second factor changes. The days change while the sales vary. Placing the factor that varies on the vertical scale as on graph A produces a graph that is easier to interpret.



Graphs C and D show how the size of the scale can affect the appearance of the graph. Graph D has larger spaces between the vertical scale measurements than the horizontal measurements, and so the graph makes the changes in sales appear steeper or larger.





Exercises

1. Use the following data chart to construct a line graph. The variable number of calls are to be on the vertical scale. Be sure to label the vertical and horizontal scales and give the graph a title.

Emergency Calls	
Time	Calls Received
6 A.M.	65
8 A.M.	70
10 A.M.	80
12 NOON.	90
2 P.M.	95
4 P.M.	95
6 P.M.	90

2. Construct a bar graph to record the information about the types and numbers of classroom pets. Use the horizontal scale for the number of pets and be sure to start at 0.

Classroom Pets	
Pet	Number
White mice	4
Hamsters	2
Guinea pigs	3
Guppies	10
Snake	1

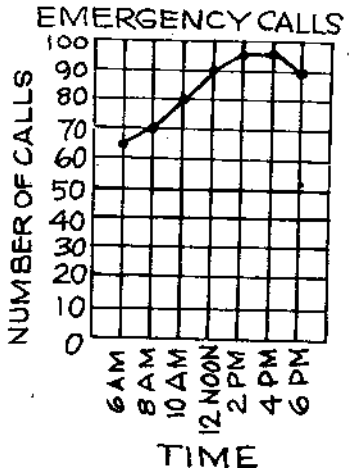
Did You Know?

A pictograph using an X to represent 10 points is often used by domino players. Half of the X, a single slash, indicates 5 points. Examples of using this method are:

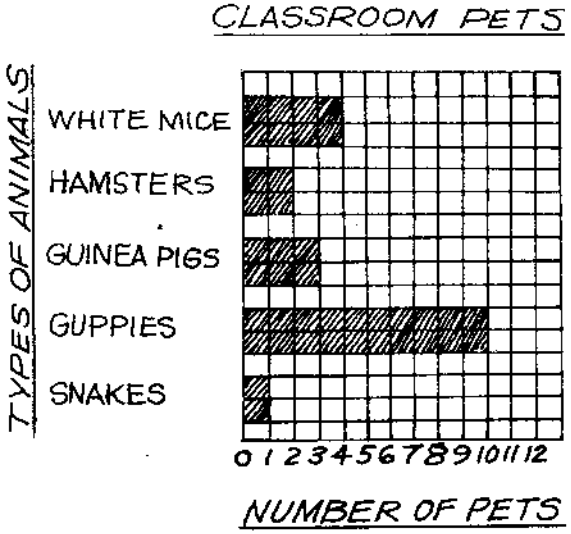
10 points	
15 points	
50 points	

Solutions

1. Though there was no time with 0 calls, the vertical scale starts at 0 calls. There is a better comparison when the vertical scale begins at zero.



2.



3. ***Think!*** What is the total number of beads?

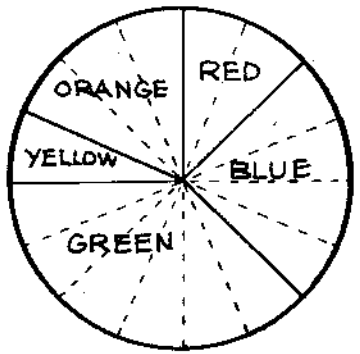
$8 + 16 + 24 + 4 + 12 = 64$

What is the smallest number? 4

$4 \times ? = 64$

$? = 16$

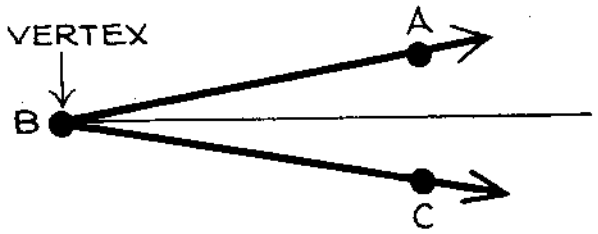
Divide the circle into 16 parts and each part will be equal to 4 beads.



26
Angles

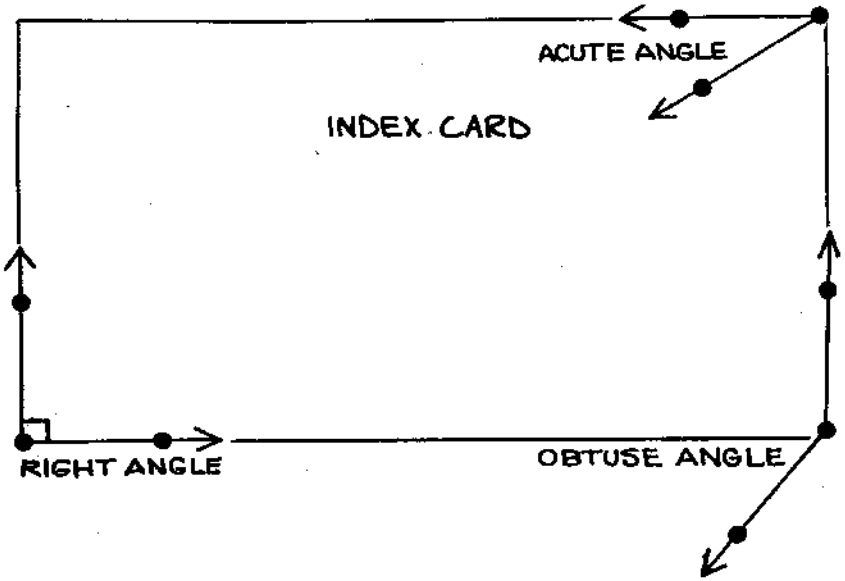
Purpose To name and identify right, acute, and obtuse angles.

Facts A **ray** is a straight line with one endpoint. An **angle** is formed when two rays have the same endpoint. The endpoint is called the **vertex**. Three letters are used to name an angle with the vertex as the center letter. Each angle has two names and either can be correctly used. The names of the angle below are: angle ABC or angle CBA. The word angle can be replaced by the symbol for the word angle, $\angle$. The names of the angle are thus $\angle$ ABC or $\angle$ CBA.



The unit of measure for an angle is a degree. One degree is written as 1°.

A **right angle** measures 90° and forms a square corner. A small square is used to indicate a right angle. An **acute angle** is any angle that measures less than 90°. An **obtuse angle** is any angle that measures greater than 90°.

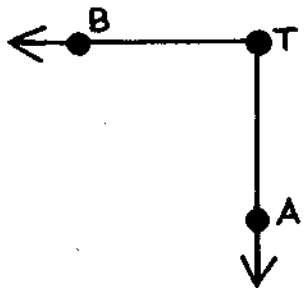


An index card can be used to identify the different angles. Place the card inside the angle so that one edge lies on a ray and the corner of the card points the angle's vertex. A right angle has both rays in line with the edge of the card. An acute angle has one ray under the card. An obtuse angle has one ray pointing away from the card.

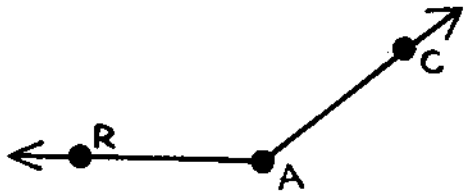
Problem

Question Use an index card to identify each angle as right, acute, or obtuse.

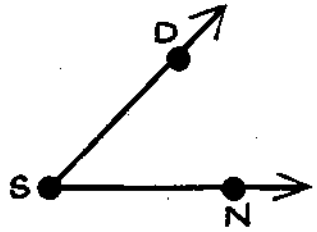
a.



b.



c.



a. $\angle BTA$ or $\angle ATB$, right angle

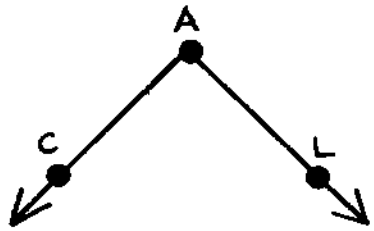
b. $\angle RAC$ or $\angle CAR$, obtuse angle

c. $\angle DSN$ or $\angle NSD$, acute angle

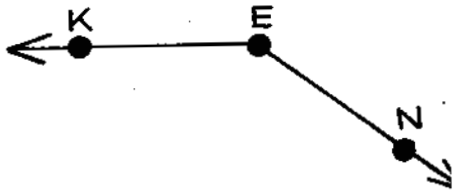
Exercises

1. Use an index card to identify each angle as right, acute, or obtuse.

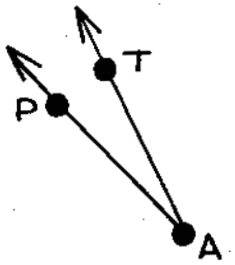
a.



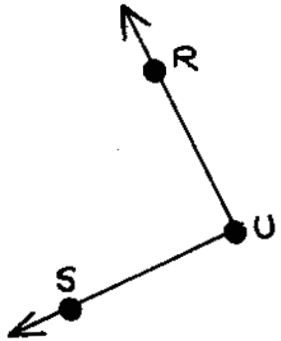
b.



c.



d.



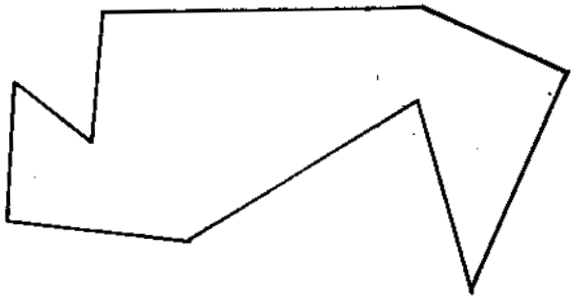
2. Draw an example for each of the angles.

   a. ∠KIM, a right angle

   b. ∠RED, an acute angle

   c. ∠MEG, an obtuse angle

3. Find the number of right, acute, and obtuse angles formed by the sides of this irregular-shaped figure.



Activity: PUSH

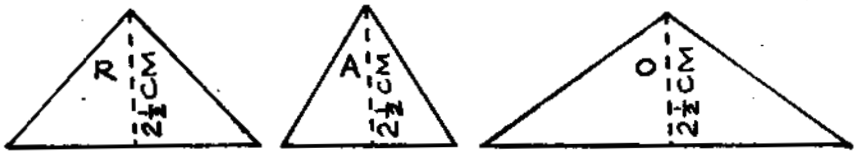
Purpose To demonstrate how the angle of a boat's bow affects its movement.

Materials

cookie sheet	toothpick
stiff paper, use a file folder	dish soap
ruler	pencil
scissors	two helpers

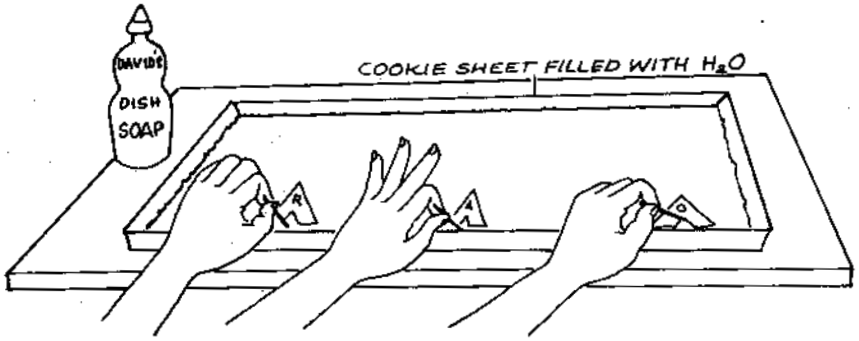
Procedure

- Draw three triangles about 1 in. (2½ cm) high on the stiff paper.
- The top of one triangle is to be a right angle; the top of the second angle is to be acute; and the top of the third angle is to be obtuse.



- A small notch is to be cut out of the base of each triangular-shaped boat.
- Fill the cookie sheet with water.

- Space the paper boats on the surface of the water at the edge of the pan.



- Moisten the tips of three toothpicks with dish soap and give one to each of your two helpers.
- Each of you is to touch the water inside the notch of a triangle boat with the wet toothpick.
- Observe the movement of the boats.

Results The paper boats blast across the surface of the water. The boat with the acute angle should be the fastest boat and the obtuse boat the slowest.

Did You Know?

The bow of a boat is pointed to allow it to easily move through the water. Cars are designed to cut through the air as they move, while large trucks often have a high, curved shield attached to divert the air up and away. The less air and water resistance for the cars, trucks, and boats, the less fuel needed to move the vehicle. This is one small way to be environmentally savvy.

Solutions

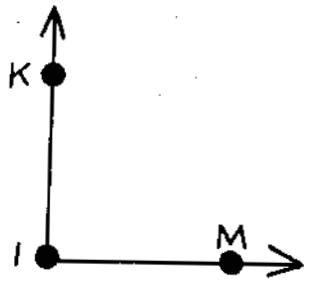
1. **a.** Right angle

   b. Obtuse angle

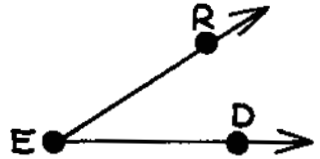
   c. Acute angle

   d. Right angle

2. **a.**



b.



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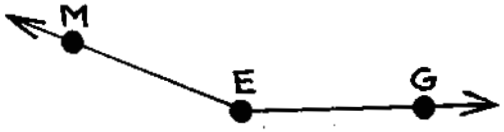
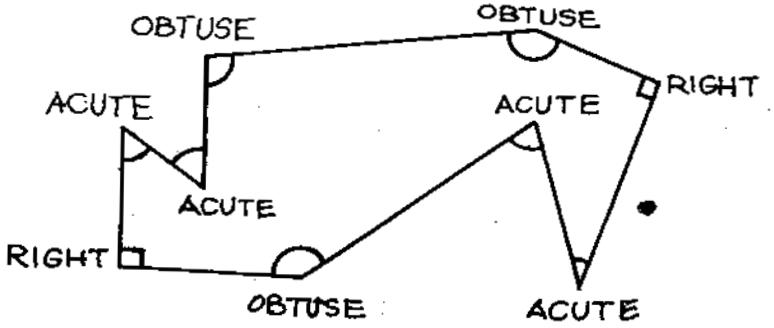


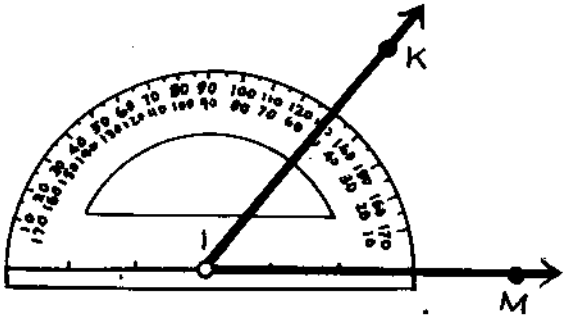
3.



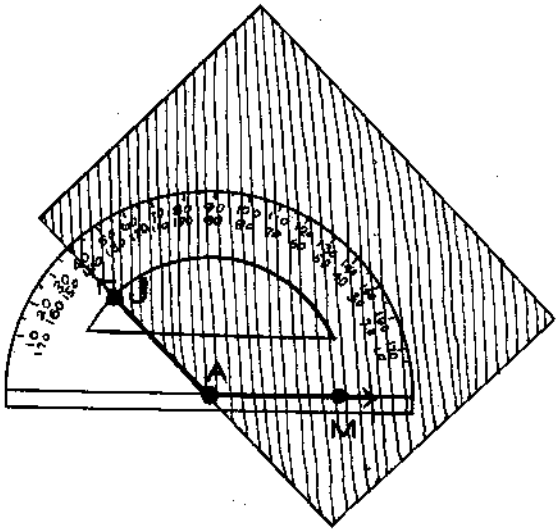
27
Protractor

Purpose To measure angles using a protractor.

Facts A **protractor** is an instrument used to measure angles in degrees. It is shaped like a half-circle. To measure an angle with a protractor, place the center mark of the instrument on the vertex of the angle and the edge on one ray of the angle. There will be two numbers to choose from where the second ray crosses the scale. One of the numbers will represent an acute angle (one that is less than 90°), and the other will be obtuse (more than 90°). The sum of these two numbers will always equal 180°. The ray IK crosses the scale at 50° and 130°. Since the angel is acute, ∠KIM is 50°.

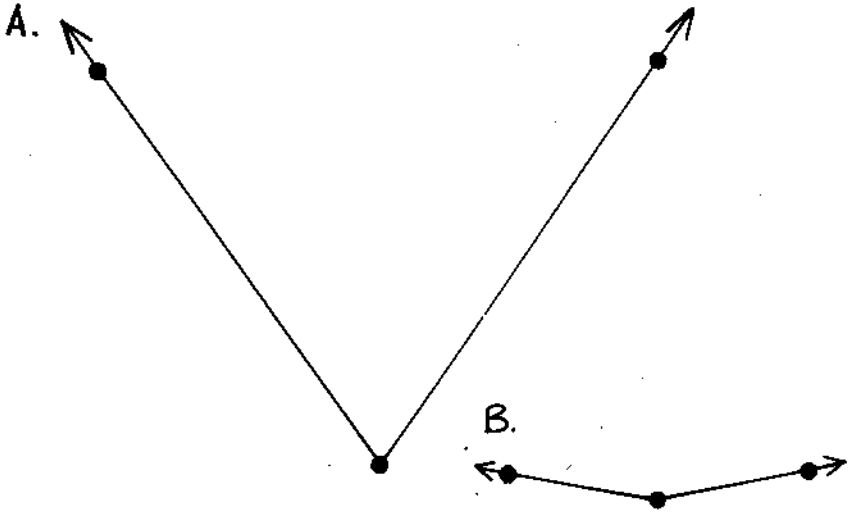


A sheet of paper or any straight edge can be used when angles have rays too short to cross the scale of the protractor. Lay the edge of the paper along the ray and read the numbers where the paper crosses the scale of the protractor. In the diagram, the edge of the paper crosses at angles 40° and 140°. Angle JAM is obtuse, so JAM is 140°.



Problem

Question Use a protractor to measure the angles in diagrams A and B.



a. ***Think!*** The angle is acute (less than 90°). Which of the angles choices, 110° or 70°, is acute?

Answer 70°

b. ***Think!*** The angle is obtuse (greater than 90°). Which of the angle choices, 160° or 20°, is obtuse?

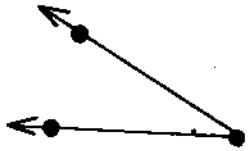
Answer 160°

Exercises

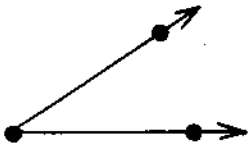
1. Use a protractor to measure the angles.

a.

b.

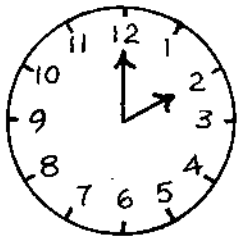


c.

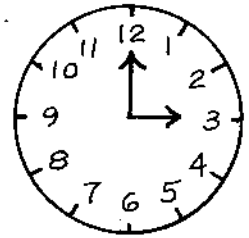


***2.** The face of a clock can be used to give direction. If straight ahead is defined as North and the large hand points to the hour of 12, what is the direction for these times?

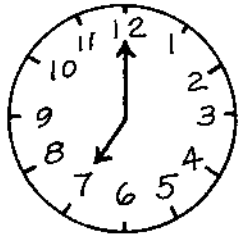
a. 2 o'clock



b. 3 o'clock



c. 7 o'clock



Activity: SUN CLOCK

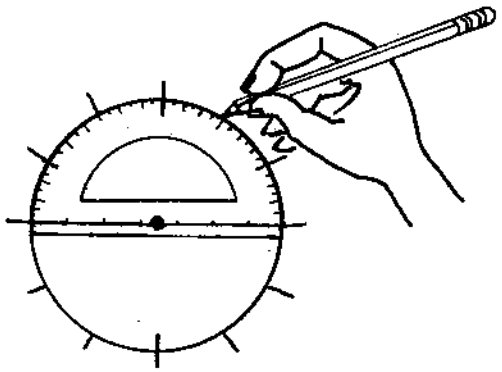
Purpose To use a protractor to make a sun clock.

Materials

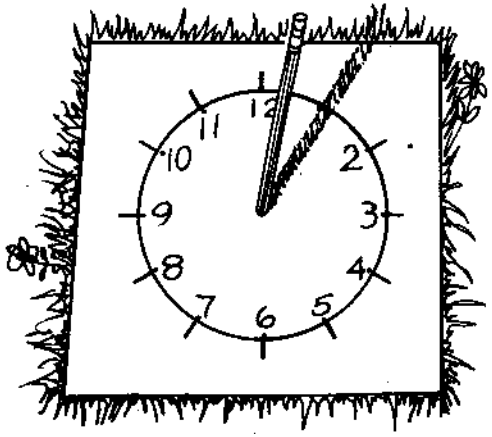
piece of cardboard, about 8 in. (20 cm) square	
protractor	marking pen
pencil	watch

Procedure

- Place the protractor on a piece of cardboard.
- Use the marking pen to draw around the outside of the protractor.
- Mark the positions of these angles on the cardboard:
  0°, 30°, 60°, 90°, 120°, 150°, 180°.
- Turn the protractor around and mark around the outside of the protractor to complete the circle.



- Mark the positions of these angles on the cardboard:
  30°, 60°, 90°, 120°, 150°.



- Write the numbers 1 through 12 on the inside of the circle as they appear on the face of a clock. Position each number beneath one of the marked angles.
- Place the cardboard on the ground with the clock diagram facing up. Be sure the clock will be in direct sunlight.
- Insert the point of the pencil through the center of the clock and into the ground. The pencil must stand straight up and down.
- When your watch reads 1:00, rotate the cardboard around the pencil so that the shadow of the pencil falls across the number 1 on the circle. (Set your watch back 1 hour if you are doing this during daylight saving time.)

  Note: You may choose another time to begin your activity.
- Observe the movement of the pencil's shadow around the sun clock as time passes.

Results The numbers on the sun clock are 30° apart. The position of the sun changes as time passes. The difference in the position of the sun causes the shadow of the pencil to change. The shadow falls across the number equal to the time read on your watch.

Did You Know?

The sun is not moving across the sky as it appears to do from morning to night. Instead the sun is stationary while the earth rotates on its axis toward the east.

Solutions

1. a. ***Think!*** The angle is obtuse (greater than 90°). Which of the angle choices, 40° or 140°, is obtuse?

Answer 140°

b. ***Think!*** The angle is acute (less than 90°). Which of the angle choices, 30° or 150°, is acute?

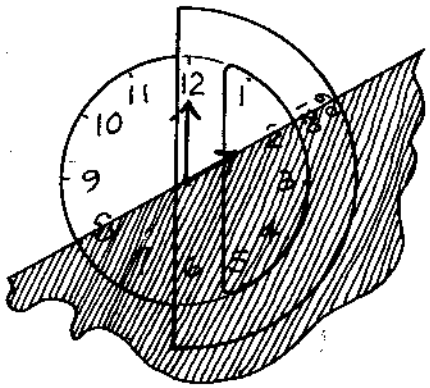
Answer 30°

c. ***Think!*** The angle is acute and touches the fifth mark between the numbered divisions. The angle choices are 35° and 145°. Which of these choices is an acute angle?

Answer 35°

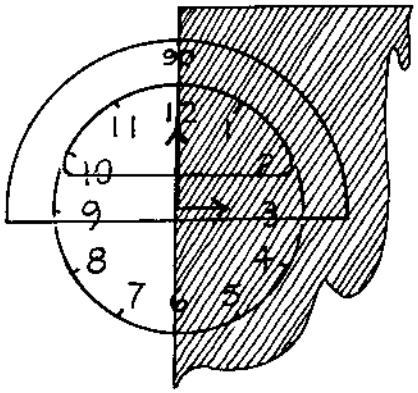
2. **a.** ***Think!*** What are the angle choices? 60° and 120°. Is the angle acute or obtuse? Acute. Is the angle to the right or left of 12? Right.

Answer 60° to the right



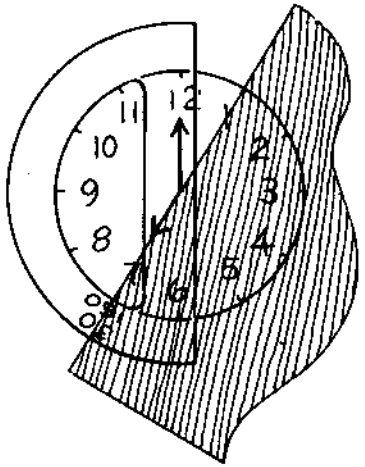
b. ***Think!*** What is the angle? 90° Is the angle to the right or left of 12? Right.

Answer 90° to the right



c. ***Think!*** What are the angle choices? 30° and 150°. Is the angle obtuse or acute? Obtuse. Is the angle to the right or left of 12? Left.

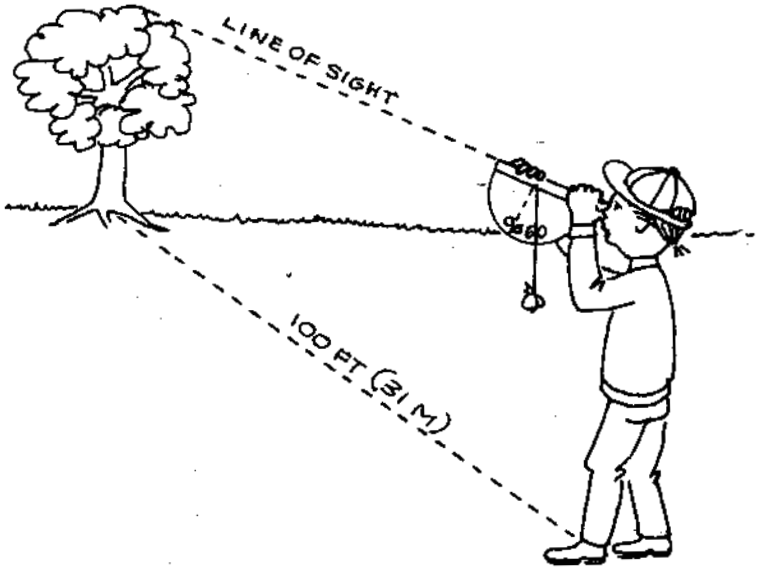
Answer 150° to the left



28
Using a Protractor

Purpose To use a protractor to determine the height of distant objects.

Facts A protractor is used in constructing an **astrolabe**, which is an instrument used to measure the heights of distant objects. In the starting position, the weighted string hangs straight down, passing over the 90° mark on the protractor scale. Tilting one end of the protractor moves the scale, but the string continues to hang straight down. The angle of the raised protractor and the Height Chart are used to determine the height of the distant object. An average height of a person using the astrolabe and an object at a distance of 100 ft (31 m) were used to determine the heights in the Astrolabe Height Chart.



Astrolabe Height Chart

Angle in Degrees	Height in Feet	Height in Meters
1	1.75	.54
2	3.52	1.08
3	5.24	1.61
4	6.99	2.15
5	8.75	2.69
10	17.63	5.43
15	26.79	8.24
20	36.40	11.20
25	46.63	14.35
30	57.74	17.76
35	70.02	21.54
40	83.91	25.82
45	100.00	31
50	119.18	36.67
55	142.81	43.94
60	173.21	53.29
65	214.45	65.98
70	274.74	84.54
75	373.21	114.83
80	567.13	174.50
85	1143.00	351.94

Problem

Question Use the readings from the astrolabe and the Astrolabe Height Chart to determine the height of the tree.

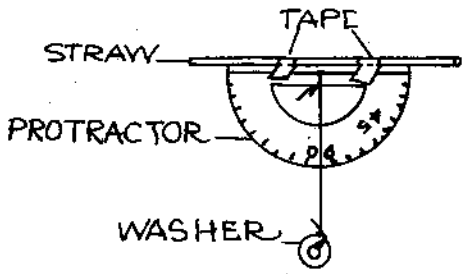
Think! On what degree did the string start? 90°

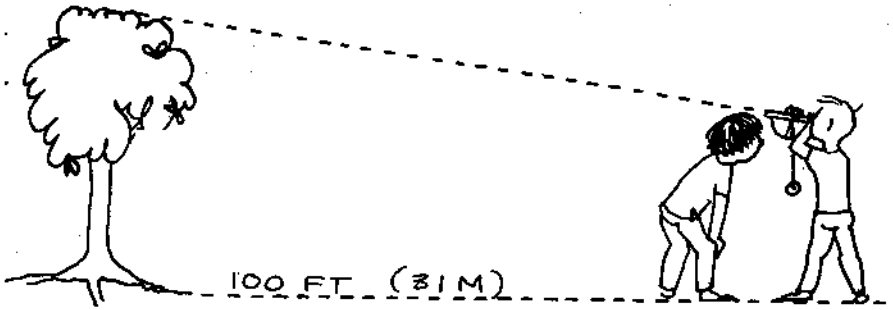
On what degree did the string stop? 60°

What is the difference between the starting and stopping position of the string? 90° − 60° = 30°

What is the height at 30°?

Answer 57.74 ft (17.76 m)





- Look through the straw at the top of the object and have your helper determine the angle of the hanging string.
- Use the Astrolabe Height Chart to determine the height of the object.

Results The angle increases as the height of the object increases.

Did You Know?

The washer continues to hang straight down while the protractor rotates because of the pull of gravity. The gravity acting on the washer is a force that pulls it toward the center of the earth.

Solutions

1. *Think!* On what degree did the string start? 90°

On what degree did the string stop? 75°

What is the difference between the starting and stopping positions of the string? $90° - 75° = 15°$

What is the height at 15°?

Answer 26.79 ft (8.24 m) is the height of the flagpole.

1. *Think!* On what degree did the string start? 90°

On what degree did the string stop? 15°

What is the difference between the starting and stopping positions of the string? $90°-15° = 75°$

What is the height at 75°?

Answer 373.21 ft (114.83 m) is the height of the rocket.

3. *Think!* On what degree did the string start? 90°

On what degree did the string stop? 40°

What is the difference between the starting and stopping positions of the string? $90° - 40° = 50°$

What is the height at 50°? 119.18 ft (36.67 m)

What is the difference between the height of the man on the rope and the height of the rope?

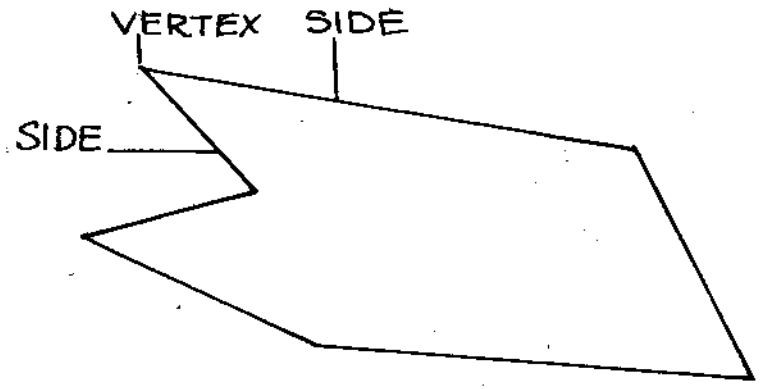
119.18 ft – 113.33 ft. = 5.85 ft
36.67 m – 34.87 m = 1.8 m

Answer 5.85 ft (1.8 m) is the height of the man.

29
Polygons

Purpose To identify polygons.

Facts A **polygon** is a simple closed figure formed by straight lines. The sides of the polygon meet to form angles. The point where the sides meet is called a **vertex**.



Polygons are named according to the number of sides in the structure. Some of the smaller and more common polygons are:

Name	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

Problem

Question For each structure, determine:

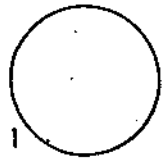
a. The number of straight sides.

b. Is it a polygon?

c. If it is a polygon, what kind of polygon.

1. **a.** No straight sides

b. It is not a polygon.



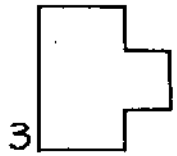
2. **a.** No straight sides

b. It is not a polygon.

3. **a.** 8 straight sides

b. Yes, it is a polygon.

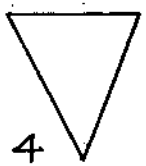
c. Octagon



4. **a.** 3 straight sides

b. Yes, it is a polygon.

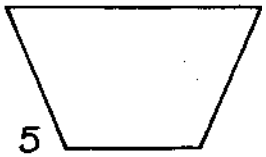
c. Triangle



5. **a.** 4 straight sides

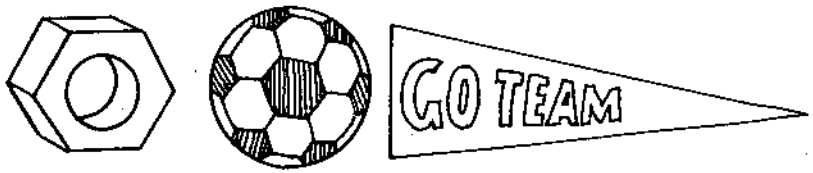
b. Yes, it is a polygon.

c. Quadrilateral



Exercises

1. Name these common polygons.

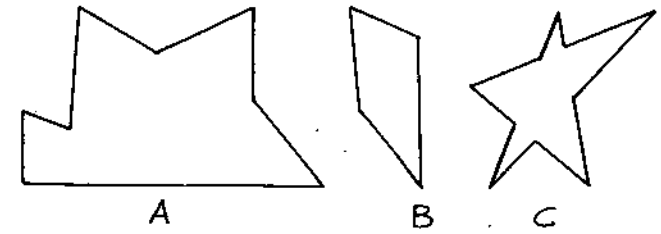


2. For each structure A through G, determine:

a. The number of straight sides.

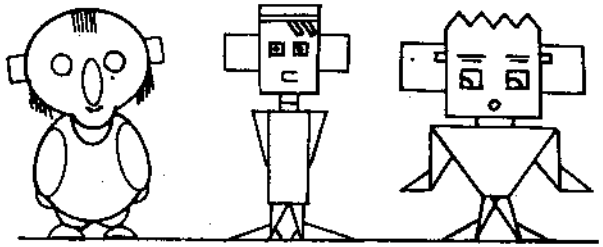
b. Is it a polygon?

c. The name of the polygon.

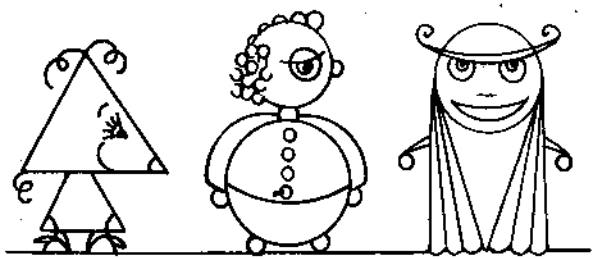


3. Study the shapes of these creatures and identify those from the indicated group that are from the imaginary planet of Zurp.

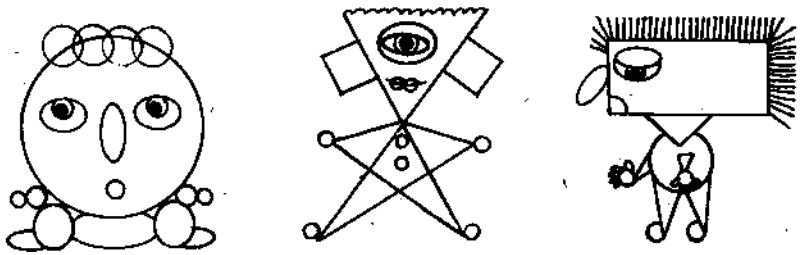
These creatures are from Zurp.



These creatures are not from Zurp.



Which of these creatures are from Zurp?



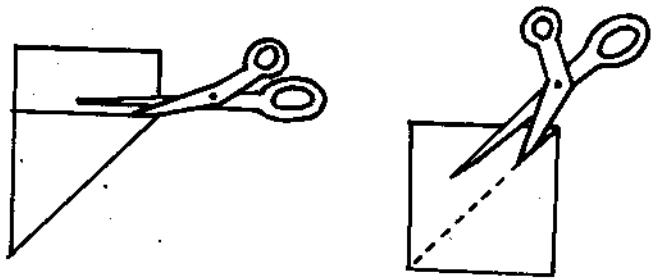
Activity: THREE TO FOUR

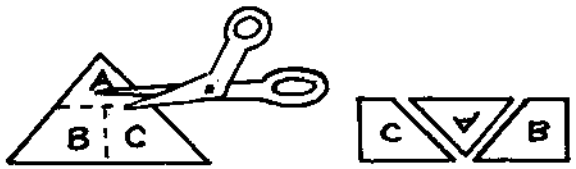
Purpose To change the shape of polygons.

Materials typing paper scissors
marking pen

Procedure

- Fold 1 sheet of paper by placing a short end against a longer side.
- Use the scissors to cut the rectangle from the end of the paper.





- Open the paper and cut across the diagonal fold. Use only one of the identical triangles that result from the cutting.
- Use the marking pen to mark a line along the edge of the triangle.
- Cut across the center of the triangle to form triangle A as seen in the diagram.
- Cut the resulting quadrilateral polygon in half to produce sections B and C.
- Position sections A, B, and C as shown to form a quadrilateral polygon.
- Rearrange the sections to form different polygons.

Results The triangle is cut and arranged to form a rectangle, a quadrilateral polygon. Many different types of polygons can be formed with the three pieces.

Did You Know?

The sum of the angles in any triangle equals 180°. The three vertices from the triangle in this activity form a straight line, 180°, when placed together.

Solutions

1. **a.** A six-sided nut is an example of a hexagon.

   b. The pattern on soccer balls is that of a pentagon.

   c. The banner is a triangle.

2. **A. a.** 8 sides

   b. Yes

   c. Octagon

   B. a. 4 sides

   b. Yes

   c. Quadrilateral

C. a. 10 sides

b. Yes

c. Decagon

D. Not a polygon

E. Not a polygon

F. a. 7 sides

b. Yes

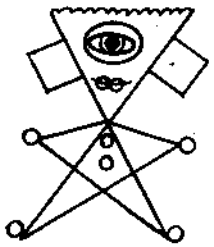
c. Heptagon

G. a. 6 sides

b. Yes

c. Hexagon

3. Zurps have square ears.



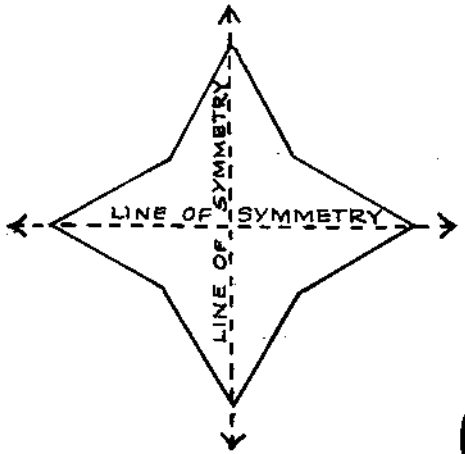
is the only Zurp.

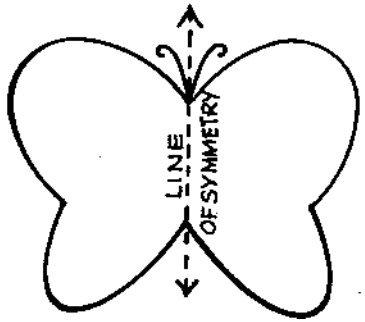
30
Symmetry

Purpose To identify lines of symmetry in a geometric figure.

Facts A **symmetric figure** can be folded along a line of symmetry and the two halves of the figure fit exactly on each other. The **line of symmetry** divides the figure into two parts that are mirror images of each other. If a mirror is placed on the line of symmetry, the whole figure can be seen.

Some figures have more than one line of symmetry as indicated by the two lines on the diagram of the star. The butterfly has only one line of symmetry.



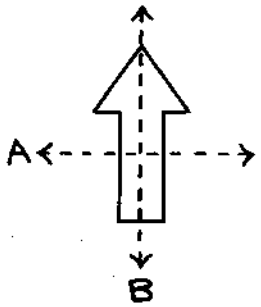


Problem

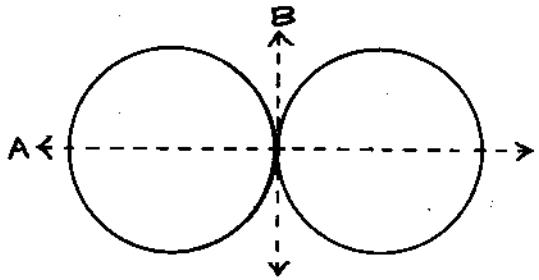
Question

Determine if the dotted lines are lines of symmetry for the figures.

a.



b.



a. ***Think!*** On which line can the figure be folded to form two halves that exactly fit together?

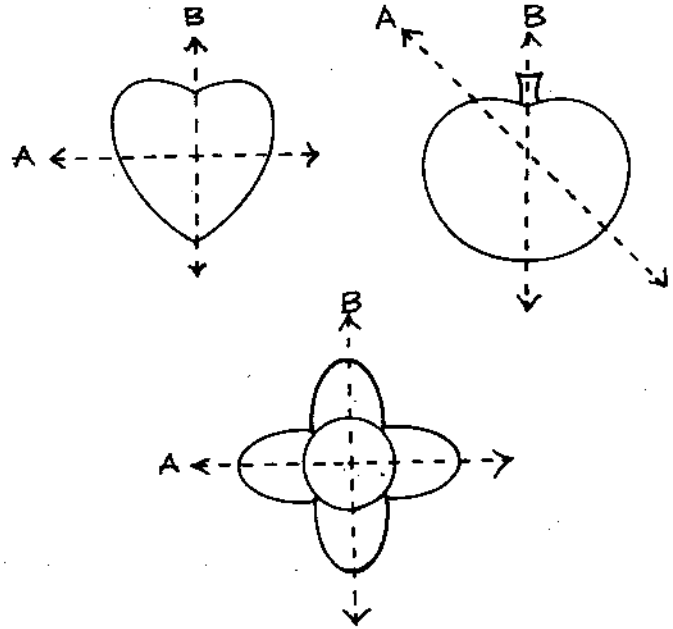
Answer Line B

b. ***Think!*** On which lines can the figure be folded to form two halves that exactly fit together?

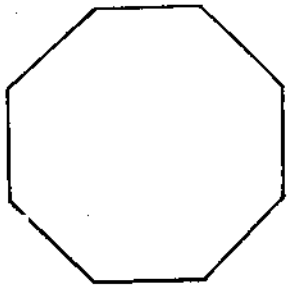
Answer Lines A and B

Exercises

1. Determine if the dotted lines are lines of symmetry for the figures.



2. How many lines of symmetry does this octagon have?



3. Take 4 sheets of paper. Fold each sheet once. On each, draw, then make a cut so that the piece has the shape when unfolded of one of these capital letters:

A, C, E, H

Activity: CUT OUTS

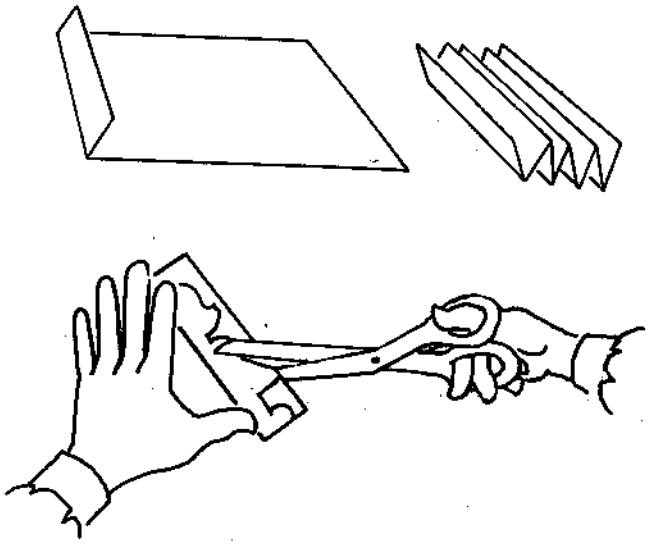
Purpose To cut out symmetrical figures.

Materials

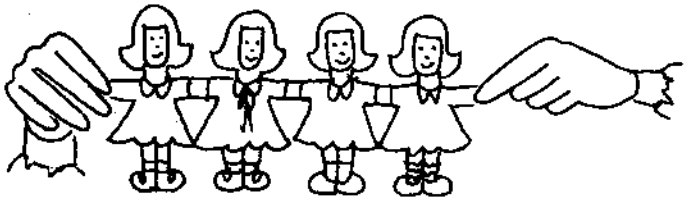
typing paper
pencil
scissors
colored marking pens

Procedure

- Fold over about 1 in. (2½ cm) of the end of 1 sheet of the paper.
- Turn the paper over and again fold the end over.
- Continue folding the ends back and forth until the entire sheet is folded like a fan.
- Flatten the folded sheet of paper.
- Use the pencil to draw half of a person on the folded paper. The left edge of the folded sheet will be the line of symmetry.
- Be sure to draw the arm out to the edge of the paper.
- Use the scissors to cut along the dotted lines. Be sure not to cut down the folded sides.



- Unfold the paper.

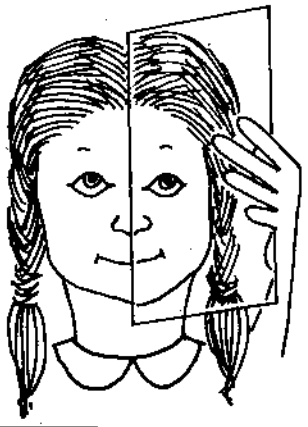


- Use the colored marking pens to add clothes and faces to the string of paper children.

Results Cutting half of a child on lines of symmetry produces a whole person. The end of the arm is also placed along a folded line of symmetry, forming a string of identical children with linking arms.

Did You Know?

Placing a mirror along the center of your face results in a whole image of your face, but the observer may be surprised to discover a new you, since your real face is not symmetrical.



Solutions

1. a. ***Think!*** On which line can the figure be folded to form two halves that exactly fit together?

Answer Line B

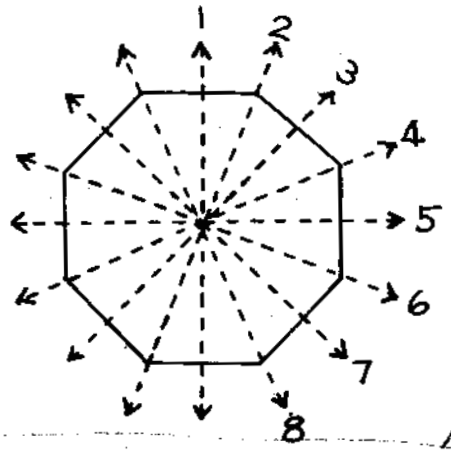
b. ***Think!*** On which line can the figure be folded to form two halves that exactly fit together?

Answer Line B

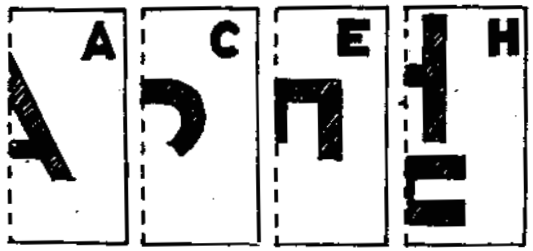
c. ***Think!*** On which lines can the figure be folded to form two halves that exactly fit together?

Answer Lines A and B

2. The octagon has 8 lines of symmetry.



3. Patterns for cutting out letters.



31 Reflections

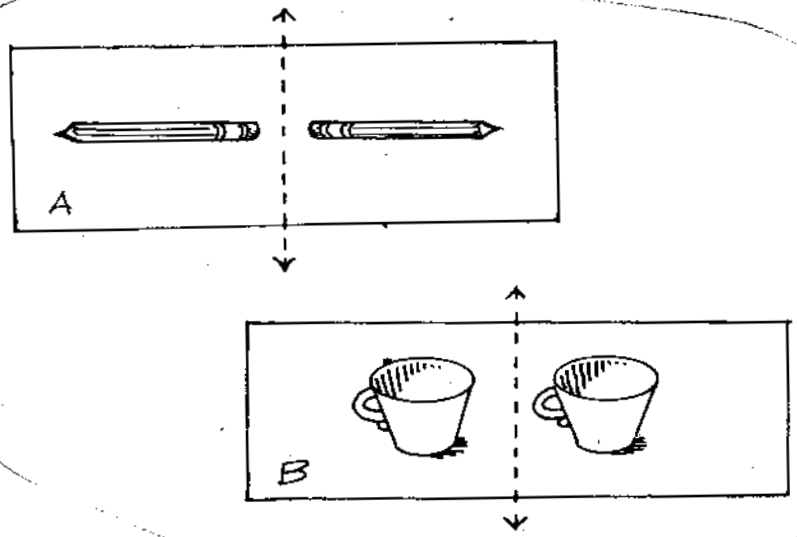
Purpose To identify mirror images.

Facts A line of symmetry divides a figure into two halves that would fit exactly on each other if folded along the line. The edge of a mirror acts like a line of symmetry between the real object and the reflected image. The real object and the reflected image would fit exactly on each other if a fold could be made at the edge of the mirror.

Problem

Question

1. Do diagrams a and b show mirror images?

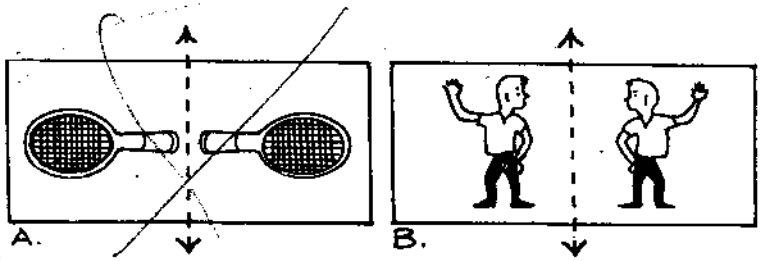


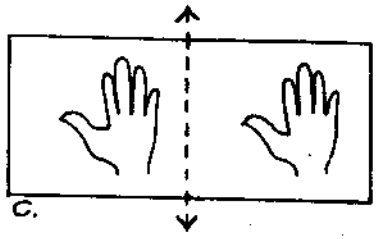
Think! Will the two halves match if the picture is folded along the dotted line?

Answer Yes, the pencils are mirror images, but the cups are *not* mirror images.

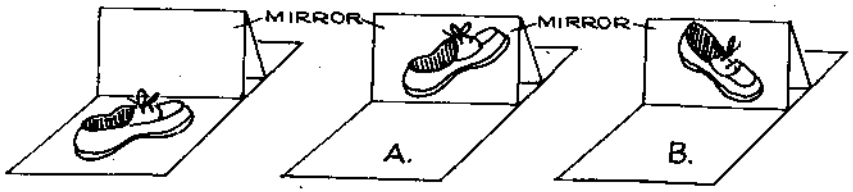
Exercises

1. Are diagrams a through c mirror images?

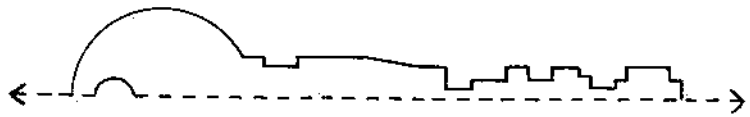




2. Placing a mirror along the dotted line completes the figure. Select the correct mirror image for the drawing.



***3.** Predict the resulting figure if a mirror is placed along the dotted line.



Activity: MIRROR IMAGES

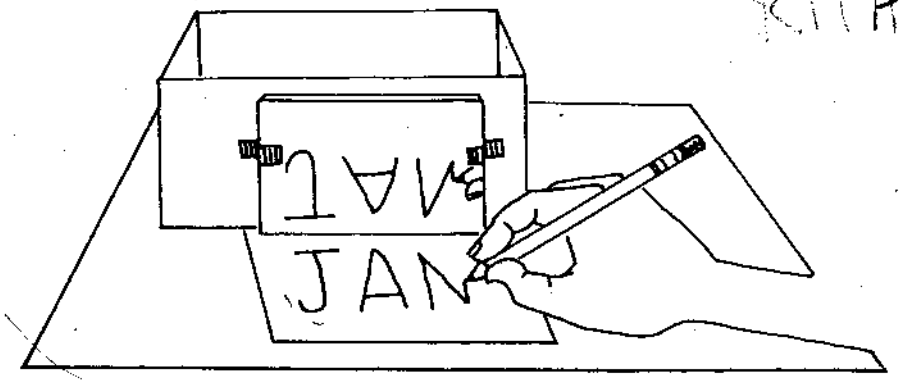
Purpose To determine the direction of mirror images.

Materials

- shoe box
- masking tape
- flat mirror
- writing paper
- pencil

Procedure

- Use the tape to secure the mirror to the side of the shoe box.
- Place a sheet of paper under the edge of the mirror.
- Use the pencil to print your name on the paper.
- Observe the direction of the letter seen in the mirror.
- Place a clean sheet of paper under the mirror's edge.
- Print your name on the paper so that the image in the mirror looks correct.



Results Mirror images are backwards. Left is right and right is left. The letters appear to be upside down because the paper is placed at an angle to the mirror. If you fold the paper at the edge of the mirror and raise it upward, the writing and the mirror image would exactly fit on each other.

Did You Know?

You have never seen yourself as others see you. Looking into a mirror always gives you a reversed image.

Solutions

1. ***Think!*** Will the two halves match if the picture is folded along the dotted line?

Answer ***a.*** Yes, the tennis rackets are mirror images.

b. Yes, the figures are mirror images.

c. No, both are right hands and are not mirror images.

2. ***Think!*** If a fold could be made at the edge of the mirror, which reflected image would match the real object?

Answer B

3. A key